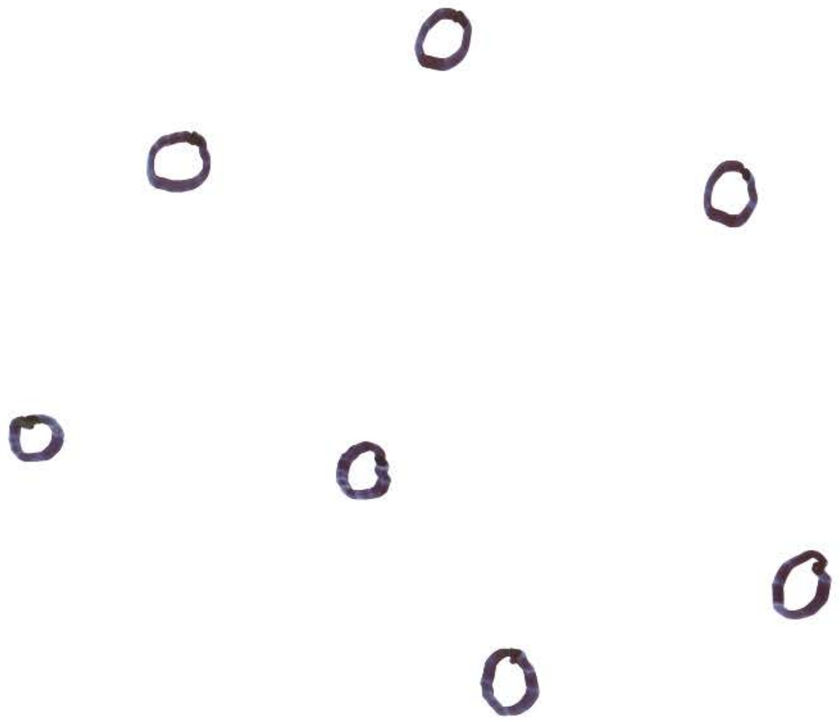
APPROXIMABILITY OF THE TSP ON POWER LAW GRAPHS

MATHIAS HAUPTMANN

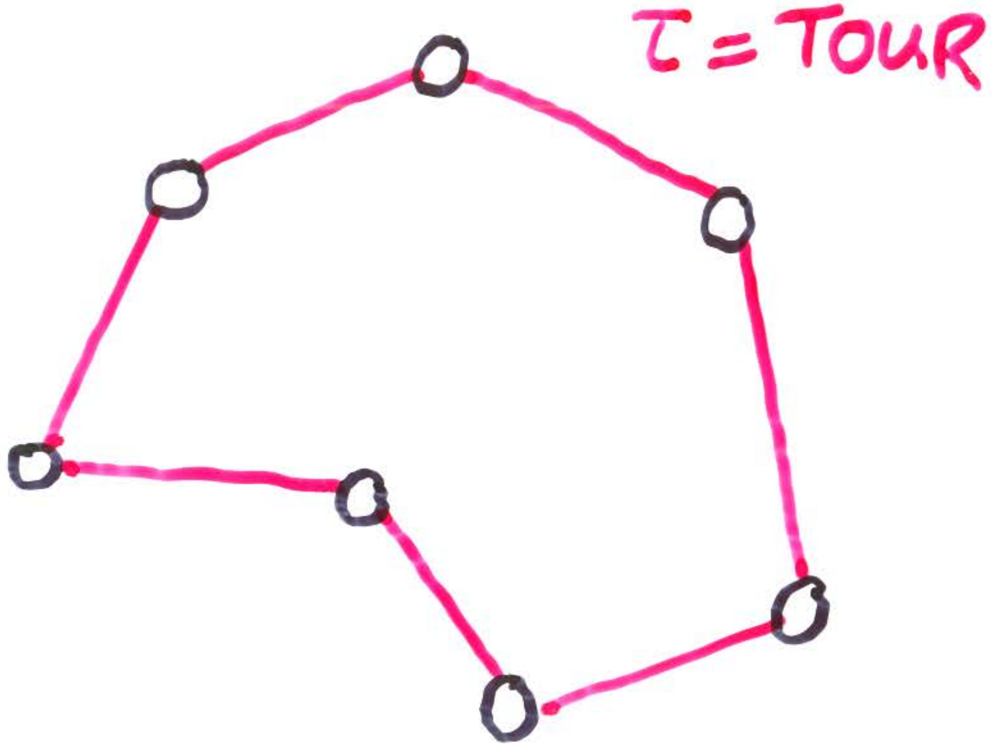
JOINT WORK WITH
MIKAEL GAST AND MAREK KARPINSKI

METRIC TSP



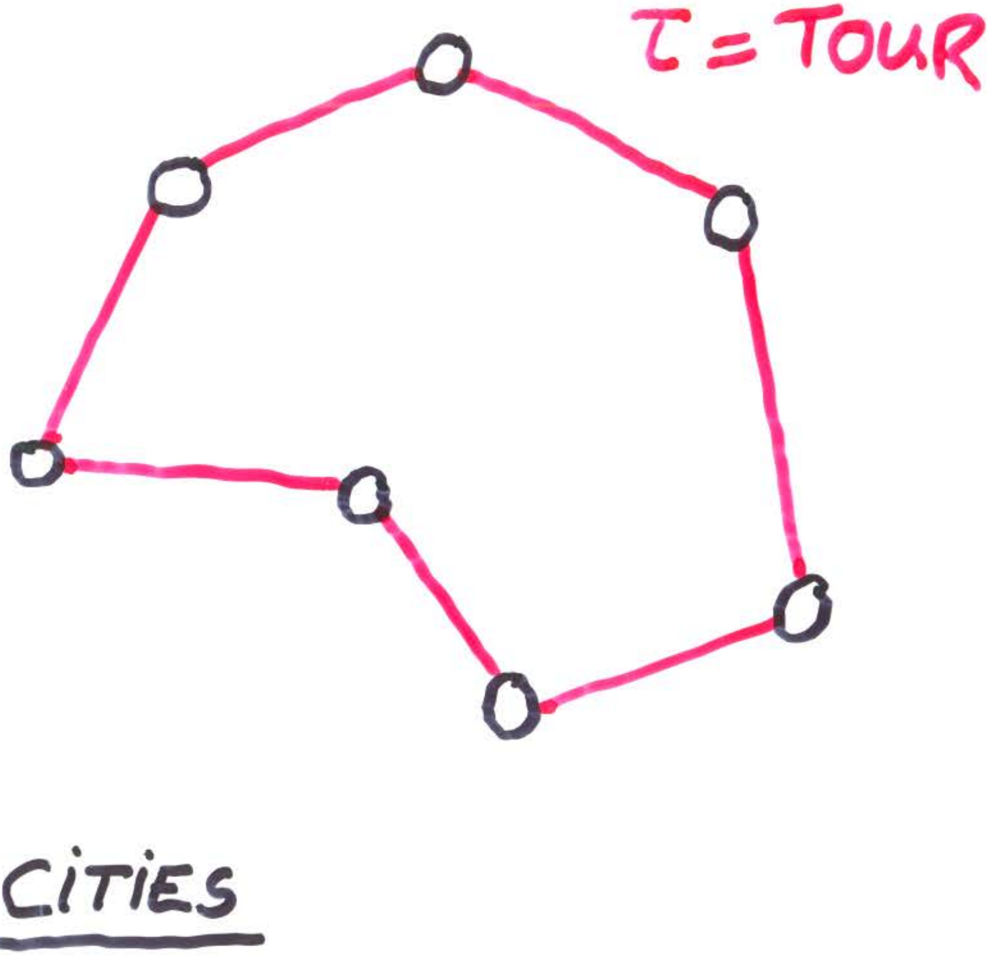
CITIES

METRIC TSP



CITIES

METRIC TSP



GIVEN:

FINITE METRIC SPACE (V, c)

FIND: TOUR τ

MINIMIZE: $c(\tau)$

STATUS OF METRIC TSP

- $\frac{3}{2}$-APPROXIMATION (CHRISTOFIDES '76)

STATUS OF METRIC TSP

- $\frac{3}{2}$-APPROXIMATION (CHRISTOFIDES '76)

- LOWER BOUND $\frac{123}{122}$

  (KARPINSKI, LAMPIS, SCHMIED '13)

RECENT IMPROVEMENTS ON SPECIAL CASES

- GRAPHIC TSP

- (1,2)-TSP

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- GRAPHIC TSP
  - (V,c) , $c =$ SHORTEST PATH METRIC OF GRAPH $G=(V,E)$

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- GRAPHIC TSP
  - (V, c) , c = SHORTEST PATH METRIC OF GRAPH $G = (V, E)$
- $(1,2)$-TSP
  - .... $c : V \times V \to \{0, 1, 2\}$

RECENT IMPROVEMENTS ON SPECIAL CASES

- GRAPHIC TSP
  - (V, c), $c =$ SHORTEST PATH METRIC OF GRAPH $G = (V, E)$

- (1,2)-TSP
  - .... $c : V \times V \to \{0, 1, 2\}$
  - .... $G = (V, E)$, $E \equiv$ PAIRS AT DISTANCE 1

STATUS OF GRAPHIC TSP, (1,2)-TSP

GRAPHIC TSP

- $7/5$-APPROXIMATION (SEBŐ, VYGEN '12)
- LOWER BOUND $\frac{535}{534}$ (KARPINSKI, SCHMIED '13)

(1,2)-TSP

- $8/7$-APPROXIMATION (BERMAN, KARPINSKI '06)
- LOWER BOUND $\frac{535}{534}$ (KARPINSKI, SCHMIED '12)

WE CONSIDER:

- GRAPHIC TSP WHERE
  $G=(V,E)$ is POWER LAW GRAPH

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- GRAPHIC TSP WHERE
  $G=(V,E)$ IS POWER LAW GRAPH

- $(1,2)$-TSP WITH G POWER LAW GRAPH

OUTLINE

1. POWER-LAW GRAPHS
2. POWER-LAW GRAPHIC TSP
3. POWER-LAW (1,2)-TSP

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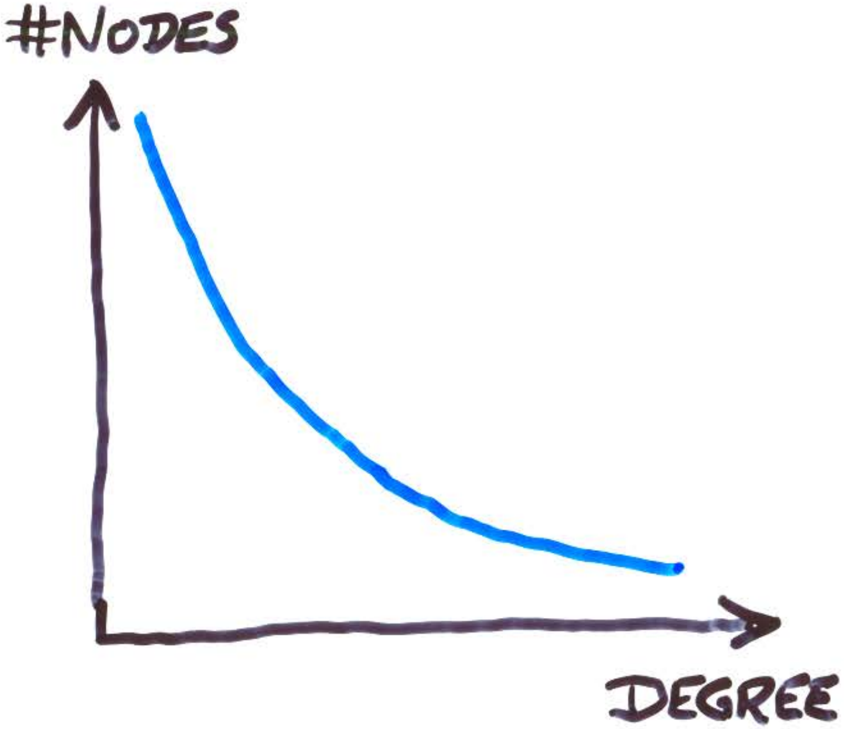
1. POWER-LAW GRAPHS
2. POWER-LAW GRAPHIC TSP
3. POWER-LAW (1,2)-TSP
    - UPPER BOUNDS
    - RANDOM INSTANCES
    - LOWER BOUNDS

OUTLINE

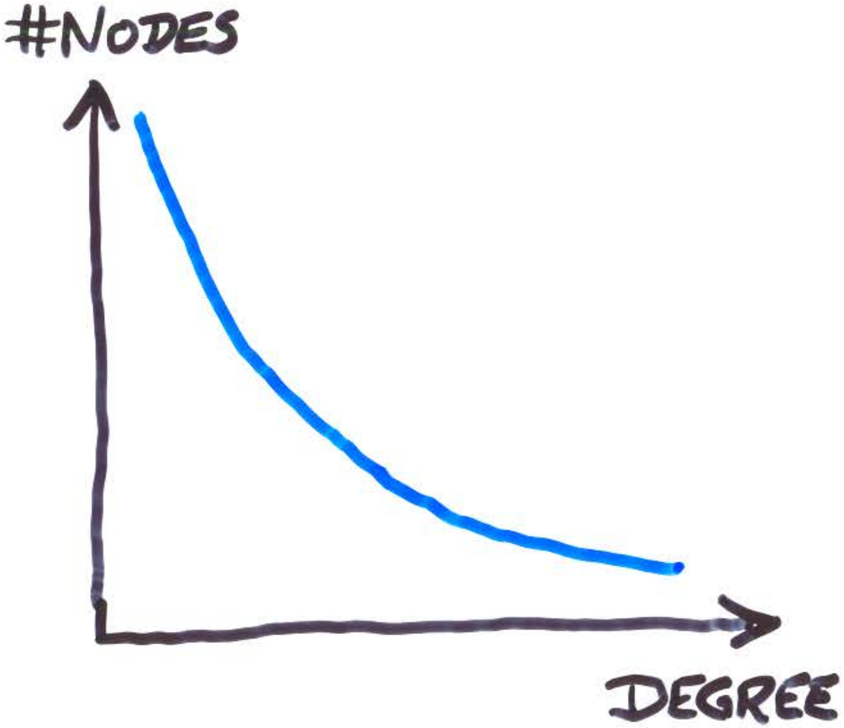
1. POWER-LAW GRAPHS
2. POWER-LAW GRAPHIC TSP
3. POWER-LAW (1,2)-TSP
   - UPPER BOUNDS
   - RANDOM INSTANCES
   - LOWER BOUNDS
4. OPEN PROBLEMS

① SCALE-FREE NETWORKS ≡ POWER-LAW GRAPHS



$$P(k) \sim k^{-\beta}$$

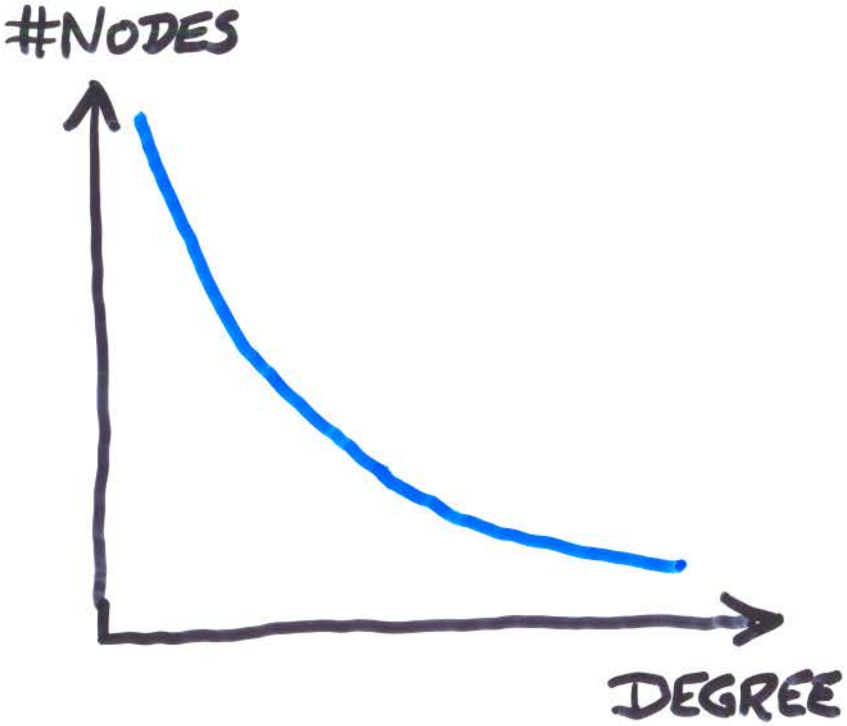
① SCALE-FREE NETWORKS
≡ POWER-LAW GRAPHS



$$P(k) \sim k^{-\beta}$$

β = POWER LAW EXPONENT

① SCALE-FREE NETWORKS $\equiv$ POWER-LAW GRAPHS

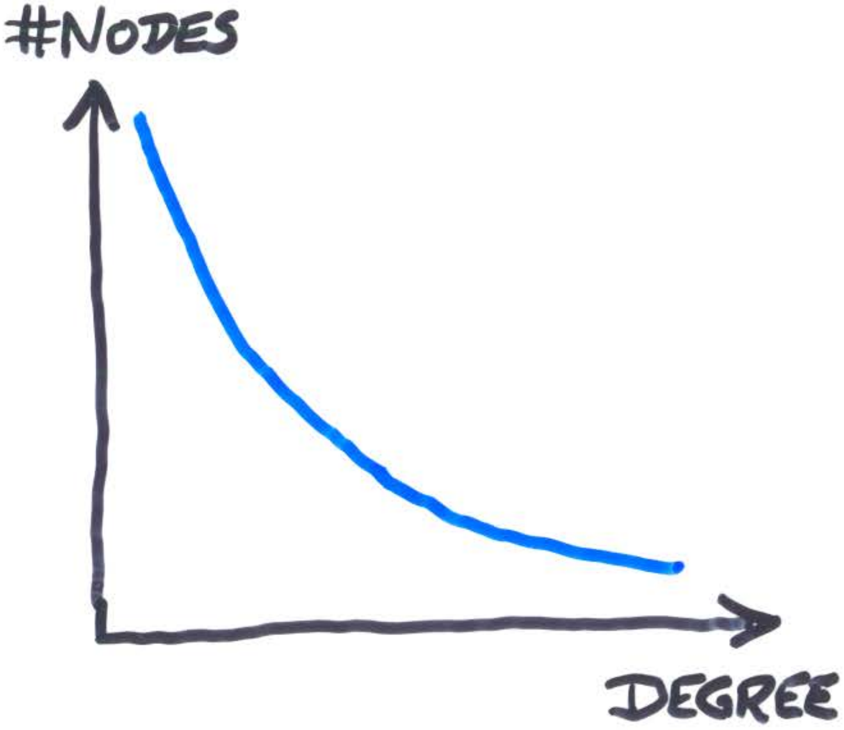


(α, β) - PLGs

$$P(k) \sim k^{-\beta}$$

β = POWER LAW EXPONENT

① SCALE-FREE NETWORKS ≡ POWER-LAW GRAPHS



$P(k) \sim k^{-\beta}$

β = POWER LAW EXPONENT

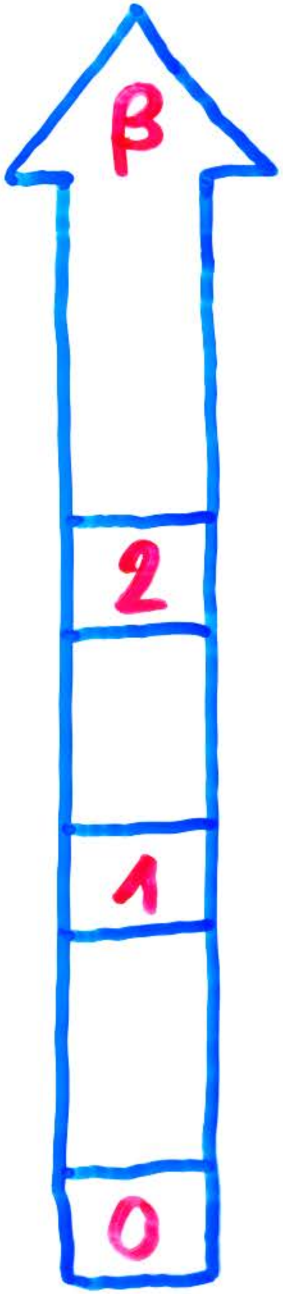
$\boxed{(\alpha,\beta)\text{-}\mathrm{PLG}_s}$

MAX DEG. $\Delta = \lfloor e^{\alpha/\beta} \rfloor$

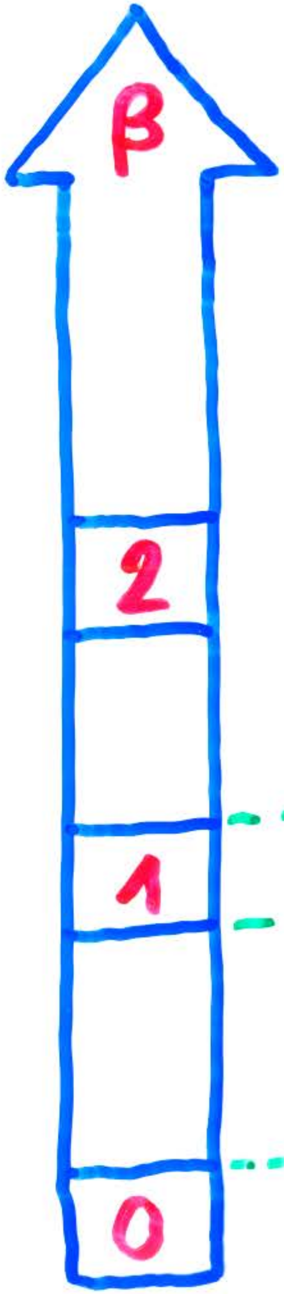
#NODES OF DEGREE i

$$y_i = \left\lfloor \frac{e^{\alpha}}{i^{\beta}} \right\rfloor$$

(α,β)-POWER LAW GRAPHS



(α,β)-POWER LAW GRAPHS



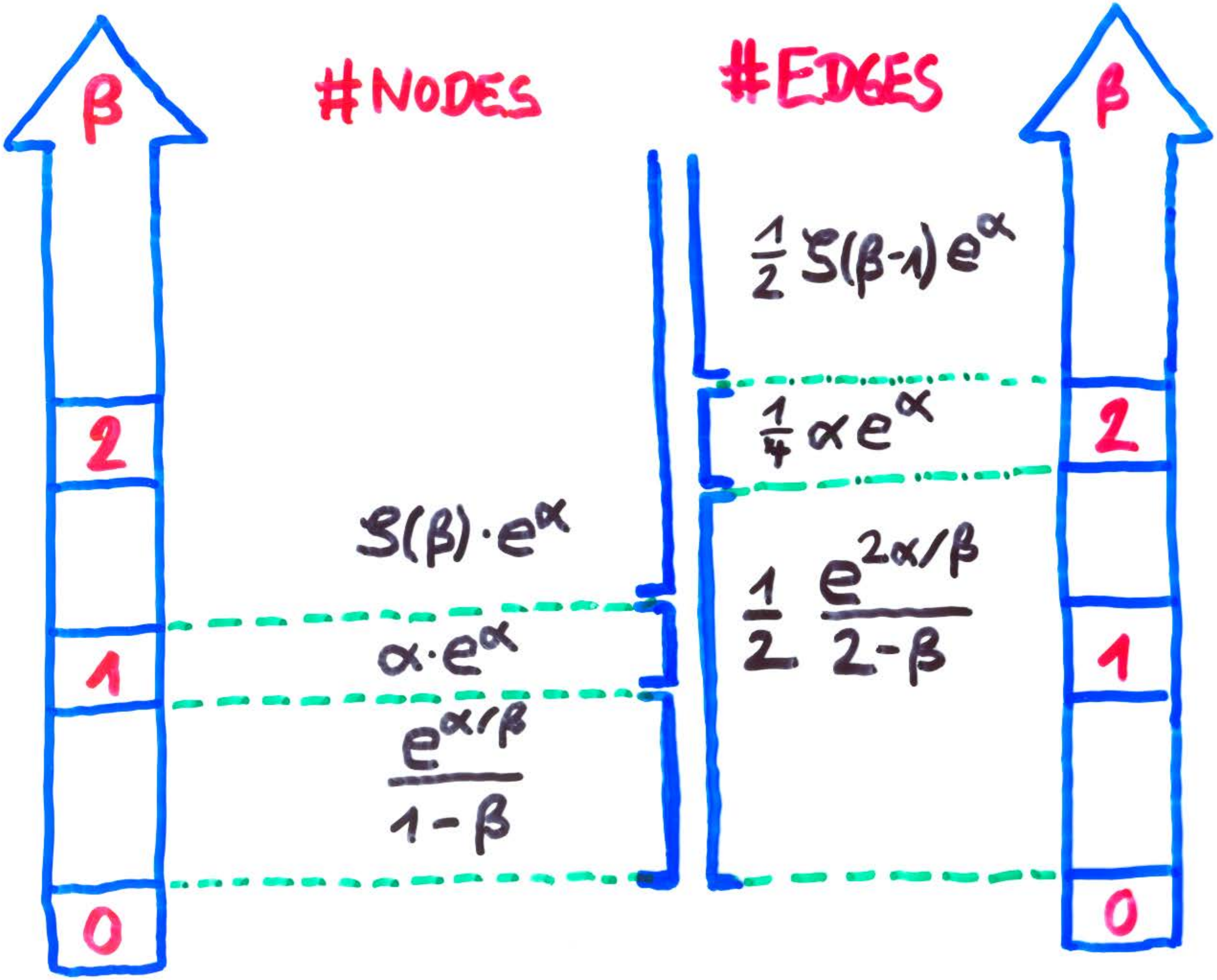
#NODES

$\zeta(\beta) \cdot e^{\alpha}$

$\alpha \cdot e^{\alpha}$

$$\frac{e^{\alpha/\beta}}{1-\beta}$$

(α,β)-POWER LAW GRAPHS



GRAPHIC TSP

- BOYD, SITTERS, VAN DER STER, STOUGIE (2011)

  4/3 FOR CUBIC GRAPHS

GRAPHIC TSP

- BOYD, SITTERS, VAN DER STER, STOUGIE (2011)

  $4/3$ FOR CUBIC GRAPHS

- MÖMKE, SVENSSON (2011)

  $4/3$ FOR SUBCUBIC GRAPHS

GRAPHIC TSP

- BOYD, SITTERS, VAN DER STER, STOUGIE (2011)
  $4/3$ FOR CUBIC GRAPHS
- MÖMKE, SVENSSON (2011)
  $4/3$ FOR SUBCUBIC GRAPHS
- MUCHA (2012): $13/9$ FOR GRAPHIC TSP

GRAPHIC TSP

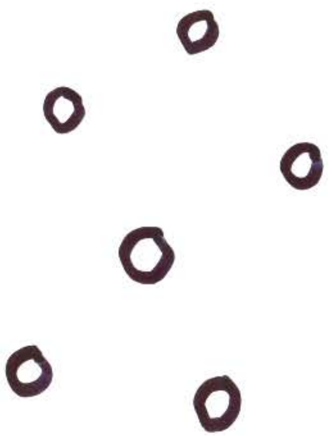
- BOYD, SITTERS, VAN DER STER, STOUGIE (2011)
  $4/3$ FOR CUBIC GRAPHS
- MÖMKE, SVENSSON (2011)
  $4/3$ FOR SUBCUBIC GRAPHS
- MUCHA (2012): $13/9$ FOR GRAPHIC TSP
- SEBŐ, VYGEN (2012): $7/5$ FOR GRAPHIC TSP

GRAPHIC TSP LOWER BOUNDS

- KARPINSKI, SCHMIED (2013)
- $\frac{535}{534}$ FOR GRAPHIC TSP
- $\frac{1153}{1152}$ FOR CUBIC GRAPHIC TSP

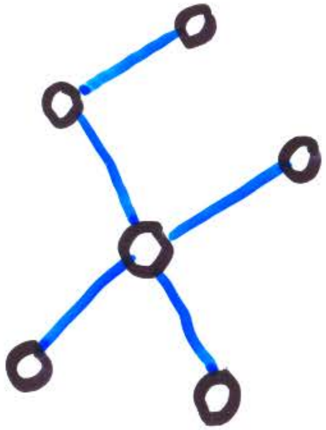
PL GRAPHIC TSP

(b) PERFORMANCE OF MST HEURISTIC



PL GRAPHIC TSP

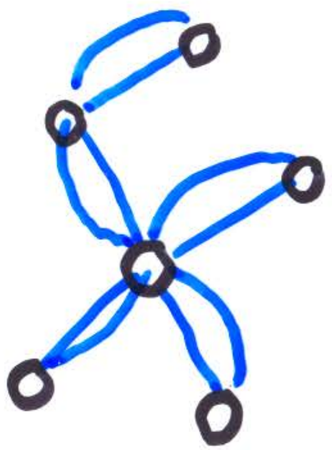
(a) PERFORMANCE OF MST HEURISTIC



MST

PL GRAPHIC TSP

(a) PERFORMANCE OF MST HEURISTIC

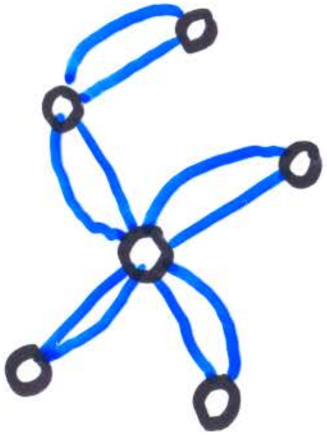


MST $\oplus$ MST

$c(MST) \leq c(\tau^*)$

PL GRAPHIC TSP

(a) PERFORMANCE OF MST HEURISTIC



MST ⊕ MST

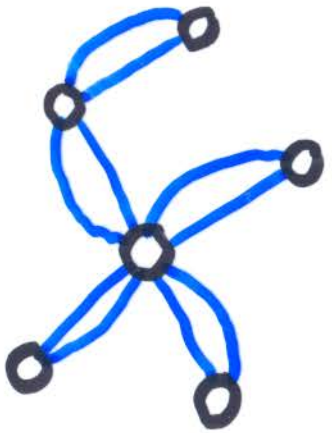
$c(MST) \leq c(\tau^*)$

<u>IN PLGs:</u>

- $c(MST) = \zeta(\beta)\, e^{\alpha} - 1$

PL GRAPHIC TSP

(a) PERFORMANCE OF MST HEURISTIC



IN PLGs:

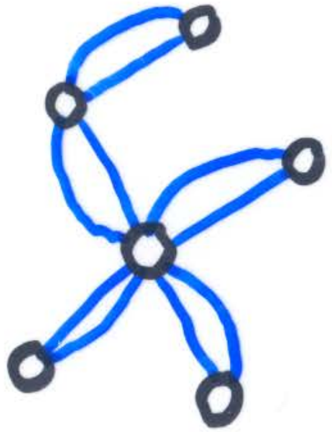
- $c(MST) = \zeta(\beta)\, e^{\alpha} - 1$
- $c(\tau^*) \geq \left(\zeta(\beta) + \frac{1}{2}\right) e^{\alpha}$

$MST \oplus MST$

$c(MST) \leq c(\tau^*)$

PL GRAPHIC TSP

(b) PERFORMANCE OF MST HEURISTIC



MST ⊕ MST

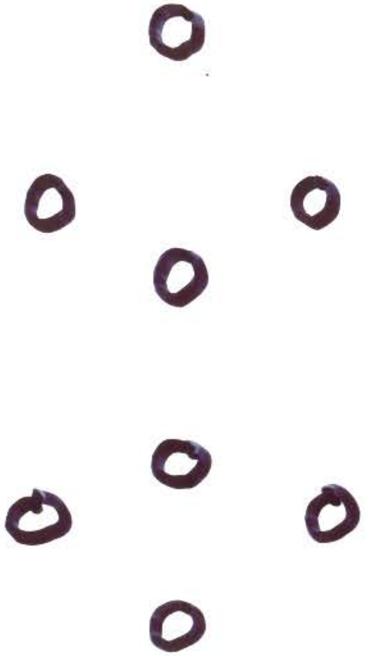
$c(MST) \leq c(\tau^*)$

IN PLGs:

- $c(MST) = \zeta(\beta)\, e^{\alpha} - 1$
- $c(\tau^*) \geq \left(\zeta(\beta) + \frac{1}{2}\right) e^{\alpha}$
- $c(\tau^*) \geq 2 \cdot e^{\alpha}$ SEE BELOW

$$\boxed{A.R(\mathcal{A}_{MST}) \leq \frac{2\,\zeta(\beta)}{\max\{2,\ \zeta(\beta) + \frac{1}{2}\}}}$$

(b) PERFORMANCE OF CHRISTOFIDES' ALG.



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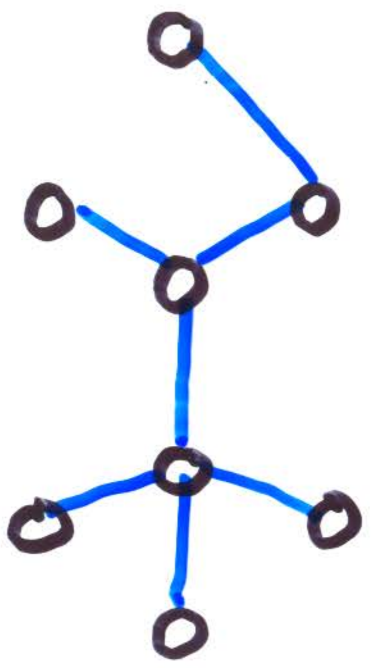
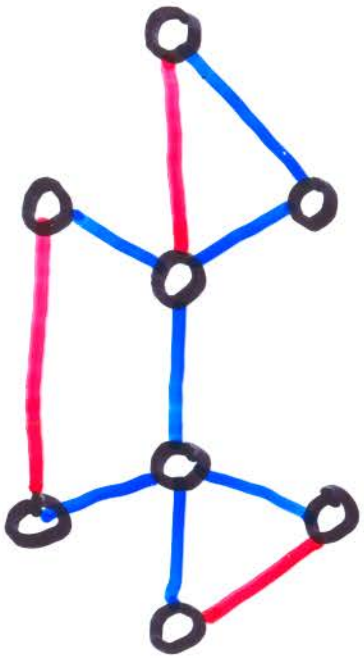


$C(MST) \leq C(\tau^*)$

(b) PERFORMANCE OF CHRISTOFIDES' ALG.

IN GENERAL: A.R $\frac{3}{2}$



$c(MST) \leq c(\tau^*)$

$c(M) \leq \frac{1}{2} c(\tau^*)$

IN (α,β)-POWER LAW GRAPHS

(b) PERFORMANCE OF CHRISTOFIDES' ALG.

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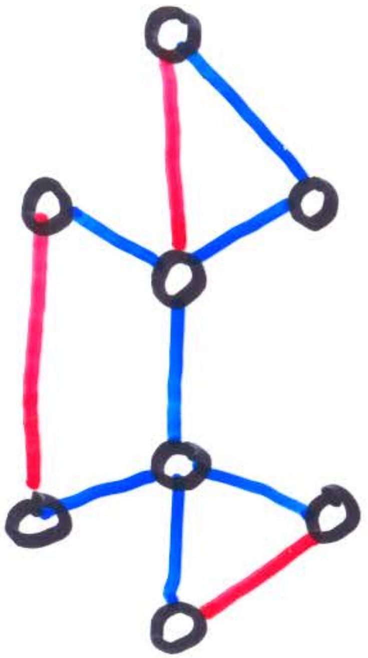


$c(MST) \leq c(\tau^*)$

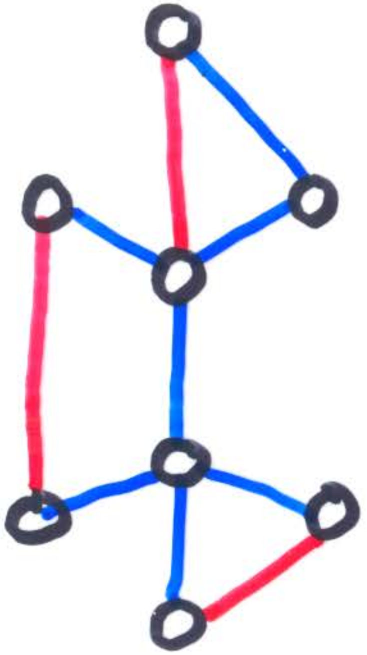
$c(M) \leq \frac{1}{2} c(\tau^*)$

IN (α, β)-POWER LAW GRAPHS

$$AR(\mathcal{A}_{CH}) \leq \frac{c(MST) + c(M)}{c(\tau^*)}$$

(b) PERFORMANCE OF CHRISTOFIDES' ALG.

IN GENERAL: A.R $\frac{3}{2}$



$c(MST) \leq c(\tau^*)$

$c(M) \leq \frac{1}{2} c(\tau^*)$

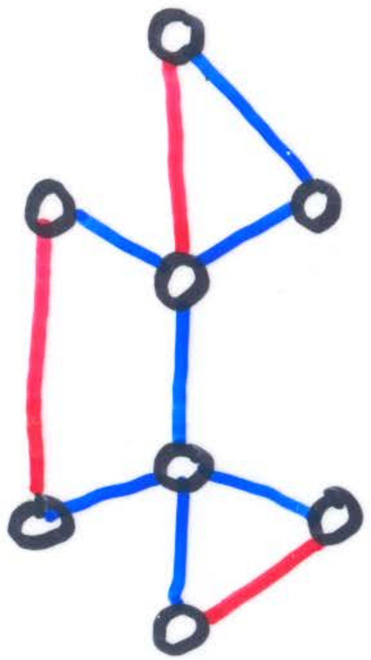
IN (α,β)-POWER LAW GRAPHS

$$AR(A_{CH}) \leq \frac{c(MST) + c(M)}{c(\tau^*)}$$

$$\leq \frac{\zeta(\beta) e^{\alpha} + c(\tau^*)/2}{c(\tau^*)}$$

(b) PERFORMANCE OF CHRISTOFIDES' ALG.

IN GENERAL: A.R $\frac{3}{2}$



$C(MST) \leq C(\tau^*)$

$C(M) \leq \frac{1}{2} C(\tau^*)$

IN (α,β)-POWER LAW GRAPHS

$$AR(A_{CH}) \leq \frac{C(MST) + C(M)}{C(\tau^*)}$$

$$\leq \frac{\zeta(\beta) e^{\alpha} + C(\tau^*)/2}{C(\tau^*)}$$

$$\leq \frac{1}{2} + \frac{\zeta(\beta)}{\text{MAX}\{2, \zeta(\beta) + \frac{1}{2}\}}$$

(c) PERFORMANCE OF MÖMKE-SVENSSON ALG. $\mathcal{A}_{MS}$ ON PLG

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$\mathcal{A}_{MS}$ BASED ON REMOVABLE PAIRINGS $(R, \mathcal{P})$

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$\mathcal{A}_{MS}$ BASED ON REMOVABLE PAIRINGS $(R,\mathcal{P})$

- $R \subseteq E$ AND $\mathcal{P} = \{P_v \mid v \in \mathcal{U}\}$ FOR SOME $\mathcal{U} \subseteq V$
- $v \in \mathcal{U}$: DEGREE ≥ 3, 

, $P_v = \{e_1, e_2\}$

(c) PERFORMANCE OF MÖMKE-SVENSSON ALG. $\mathcal{A}_{MS}$ ON PLG

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- $v \in \mathcal{U}$: DEGREE ≥ 3, 

, $P_v = \{e_1, e_2\}$
- $\forall F \subseteq R$ with $\forall v \in \mathcal{U}$ $|P_v \cap F| \leq 1$: GRAPH $(V, E \setminus F)$ CONNECTED

<u>LEMMA</u> (M-S 2011)

$\exists$ TOUR τ OF COST $\leq \frac{4}{3} c(E) - \frac{2}{3} c(R)$

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OUR RESULT FOR (α, β)-PLG :

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$\exists$ TOUR τ OF COST $\leq \frac{4}{3} c(E) - \frac{2}{3} c(R)$

OUR RESULT FOR (α, β)-PLG:

- L. BOUND ON $|R|$
- $\longrightarrow$ TOUR OF COST $\leq e^{\alpha}\left(\frac{2}{3}\zeta(\beta-1) + \frac{2}{3}\zeta(\beta) + \frac{5}{6}\right)$

LEMMA (M-S 2011)

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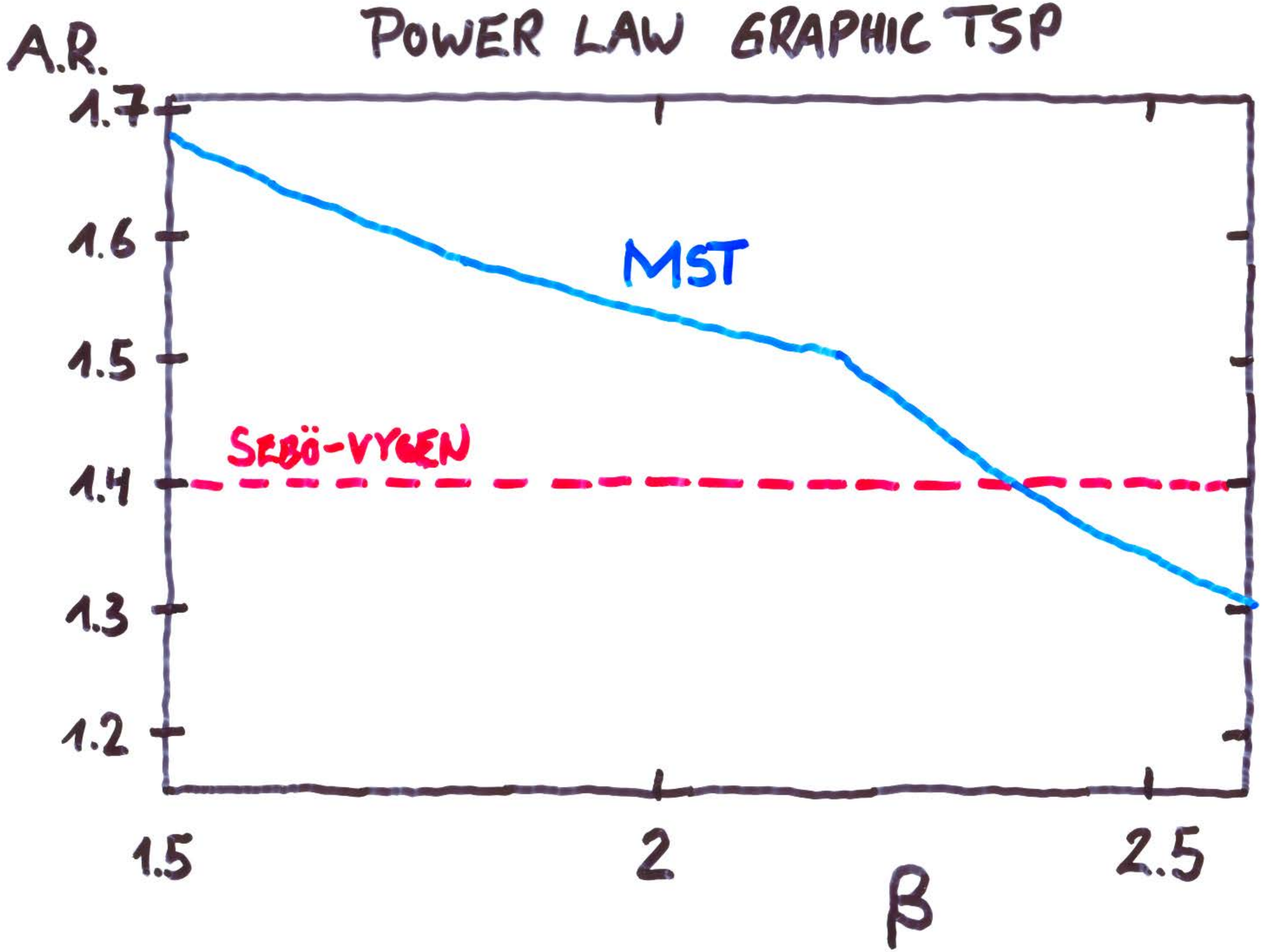
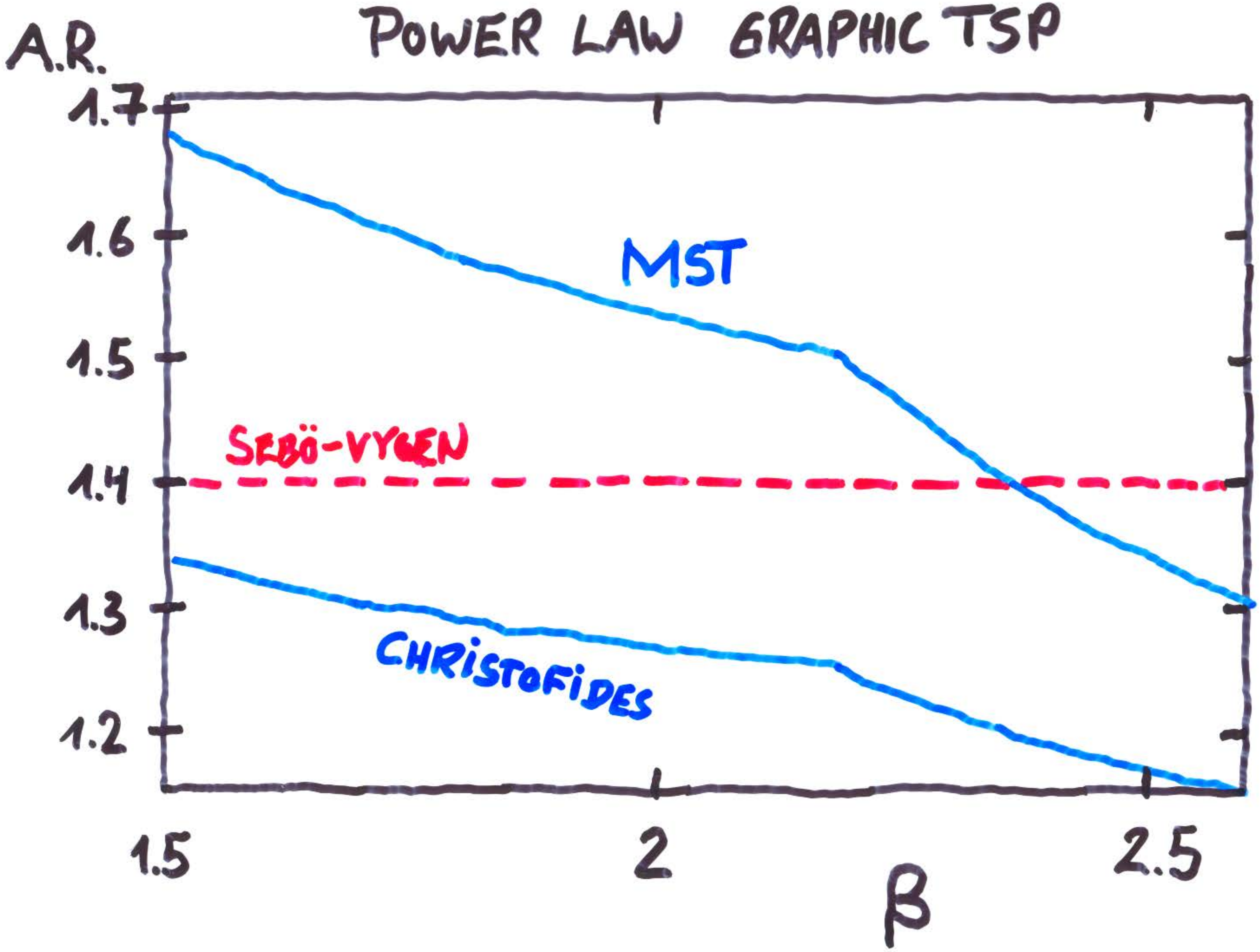
OUR RESULT FOR (α, β)-PLG:

- L. BOUND ON $|R|$
- $\longrightarrow$ TOUR OF COST $\leq e^{\alpha}\left(\frac{2}{3}\zeta(\beta-1) + \frac{2}{3}\zeta(\beta) + \frac{5}{6}\right)$
- A.R. $\dfrac{\frac{2}{3}\zeta(\beta-1) + \frac{2}{3}\zeta(\beta) + \frac{5}{6}}{\max\left\{\zeta(\beta) + \frac{1}{2}, 2\right\}}$



POWER LAW GRAPHIC TSP
A.R.
1.7
1.6
1.5
1.4
1.3
1.2
MST
SEBÖ-VYGEN
1.5
2
2.5
β


POWER LAW GRAPHIC TSP
A.R.
1.7
1.6
1.5
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1.3
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MST
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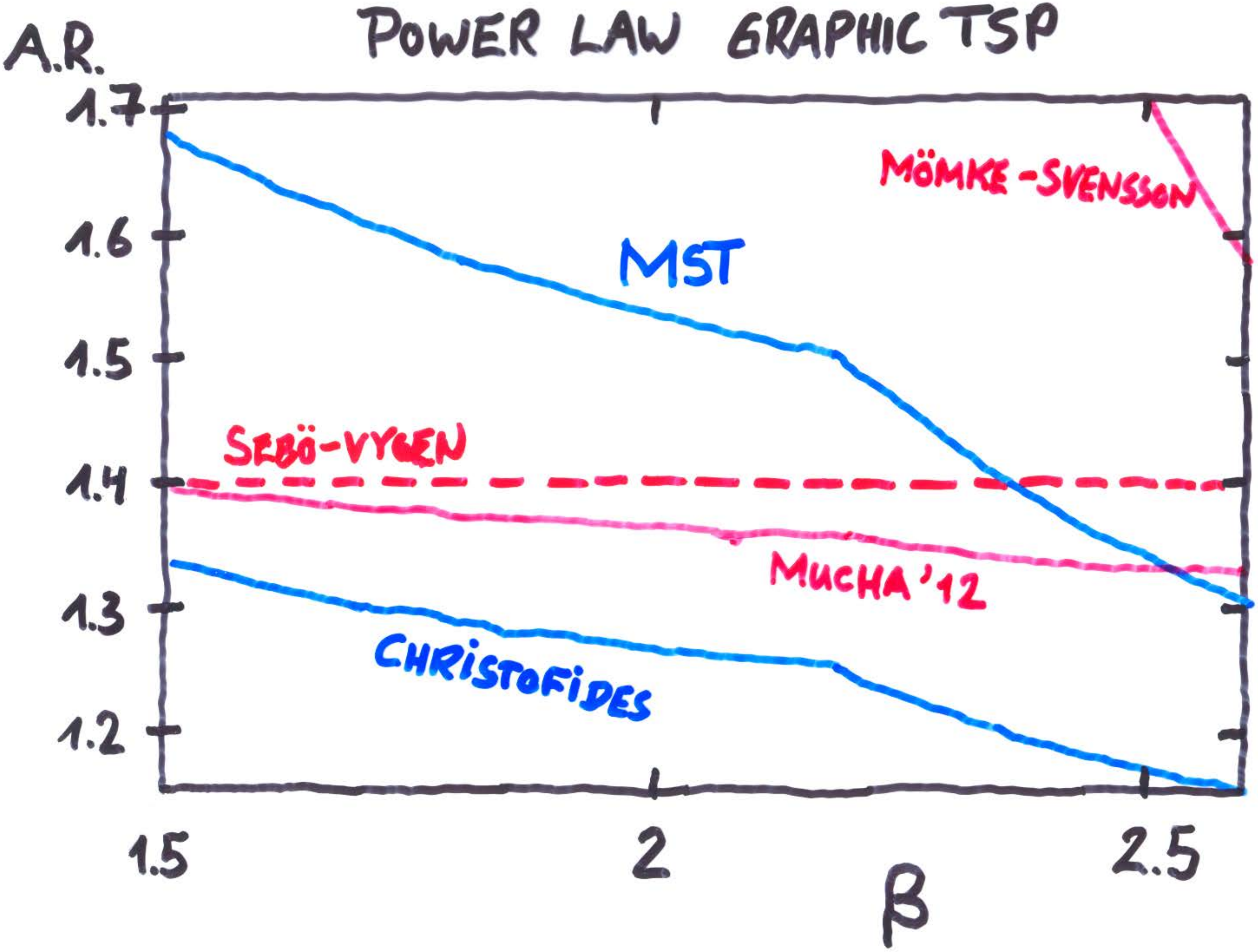
CHRISTOFIDES
1.5
2
2.5
β


POWER LAW GRAPHIC TSP
A.R.
1.7
1.6
1.5
1.4
1.3
1.2
1.5
2
2.5
β
MÖMKE-SVENSSON
MST
SEBÖ-VYGEN
MUCHA '12
CHRISTOFIDES

③ POWER LAW (1,2)-TSP

STATUS OF (1,2)-TSP

③ POWER LAW (1,2)-TSP

STATUS OF (1,2)-TSP

- $8/7$ - APPROXIMATION [BK06]

③ POWER LAW (1,2)-TSP

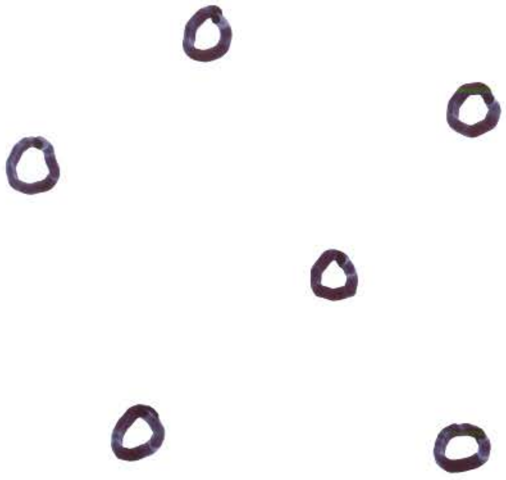
STATUS OF (1,2)-TSP

- 8/7 - APPROXIMATION [BK 06]
- APPROX. HARDNESS $\frac{535}{534}$ [KS 12]

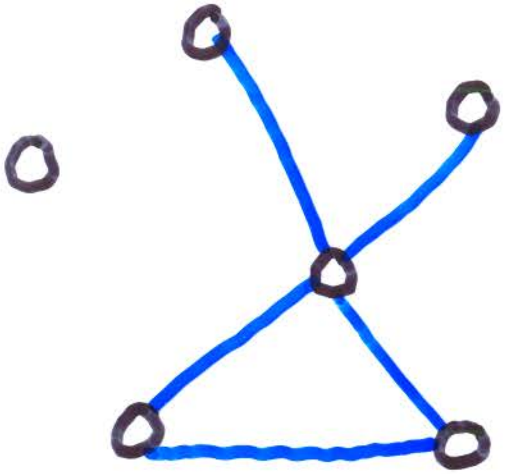
APPROXIMATION ALGORITHMS FOR (1,2)-TSP

- $7/6$ PAPADIMITRIOU, YANNAKAKIS [PY 93]
- $65/56$ BLÄSER, SHANKAR RAM [BS 05]
- $8/7$ BERMAN, KARPINSKI [BK 06]

POWER LAW (1,2)-TSP

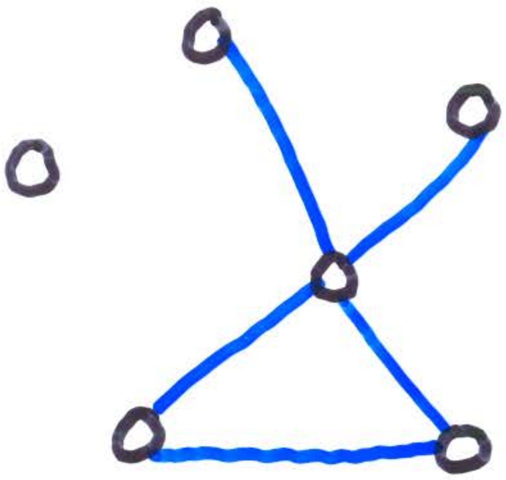


POWER LAW (1,2)-TSP



G = GRAPH OF 1-EDGES

POWER LAW (1,2)-TSP

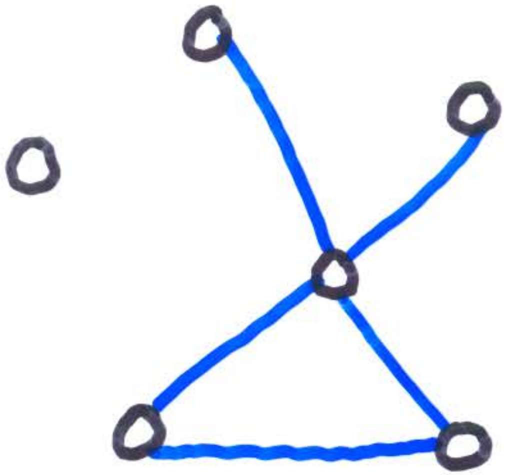


G = GRAPH OF 1-EDGES

HERE: G (α,β)-PLG

OUR RESULTS

POWER LAW (1,2)-TSP



G = GRAPH OF 1-EDGES

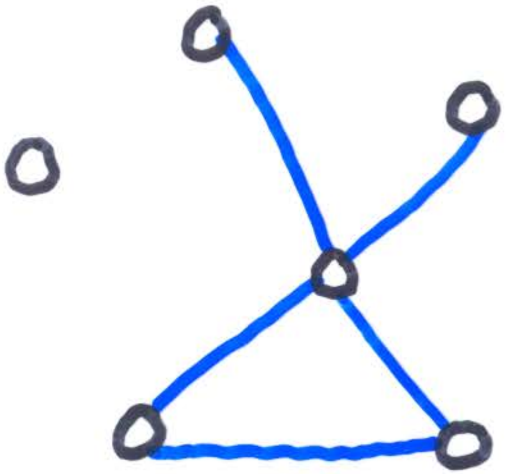
HERE: G (α, β)-PLG

OUR RESULTS

- A.R. $\dfrac{\frac{11}{9}\,\zeta(\beta) + \frac{29}{72}}{\zeta(\beta) + \frac{1}{2}}$

  FOR $\beta > 1$

POWER LAW (1,2)-TSP



G = GRAPH OF 1-EDGES

HERE: G (α,β)-PLG

OUR RESULTS

- A.R. $\dfrac{\frac{11}{9}\zeta(\beta)+\frac{29}{72}}{\zeta(\beta)+\frac{1}{2}}$

  FOR $\beta > 1$

- IMPROVED EXPECTED A.R. FOR RANDOM INSTANCES

POWER LAW (1,2)-TSP

APPROACH

POWER LAW (1,2)-TSP

APPROACH BASED ON [PY93]

- C = CYCLE COVER (2-MATCHING) WITH NO CYCLES OF LENGTH < 4

POWER LAW (1,2)-TSP

APPROACH BASED ON [PY93]

- C = CYCLE COVER (2-MATCHING) WITH NO CYCLES OF LENGTH < 4
- k = # 2-EDGES IN C

POWER LAW (1,2)-TSP

APPROACH BASED ON [PY93]

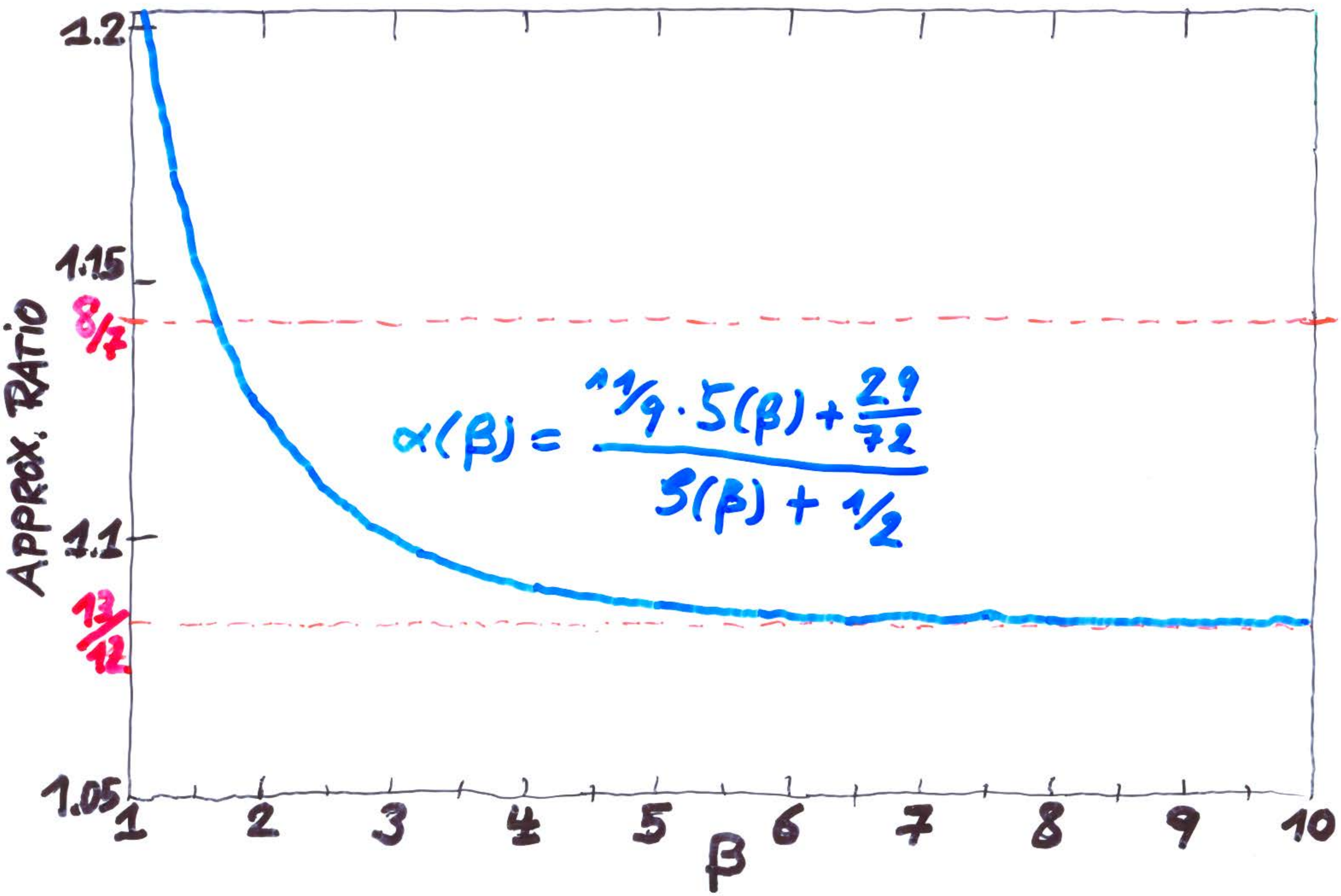
- C = CYCLE COVER (2-MATCHING) WITH NO CYCLES OF LENGTH < 4
- k = #2-EDGES IN C
- A.R. $\dfrac{\frac{11}{9}n + \left(\frac{7}{9} + \frac{1}{36}\right)k}{n+k}$

POWER LAW (1,2)-TSP

APPROACH BASED ON [PY93]

- C = CYCLE COVER (2-MATCHING) WITH NO CYCLES OF LENGTH < 4
- k = # 2-EDGES IN C
- A.R. $\dfrac{11/9\, n + (7/9 + 1/36)k}{n+k}$

▶ SHOW: FOR PLGs, k IS LARGE



1.2
1.15
8/7
1.1
13/12
1.05
APPROX. RATIO
1
2
3
4
5
6
7
8
9
10
β
$\alpha(\beta) = \frac{11/9 \cdot S(\beta) + \frac{29}{72}}{S(\beta) + 1/2}$

RANDOM PLG (1,2)-TSP

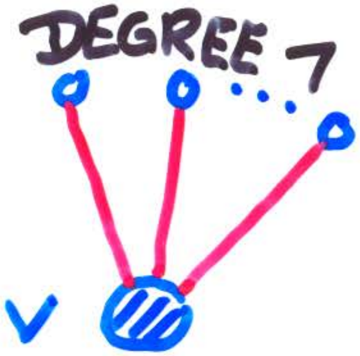
RANDOM PLG (1,2)-TSP

- EXPECTED # CONFIGURATIONS

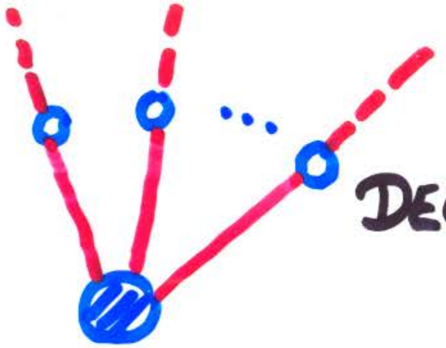
OR

RANDOM PLG (1,2)-TSP

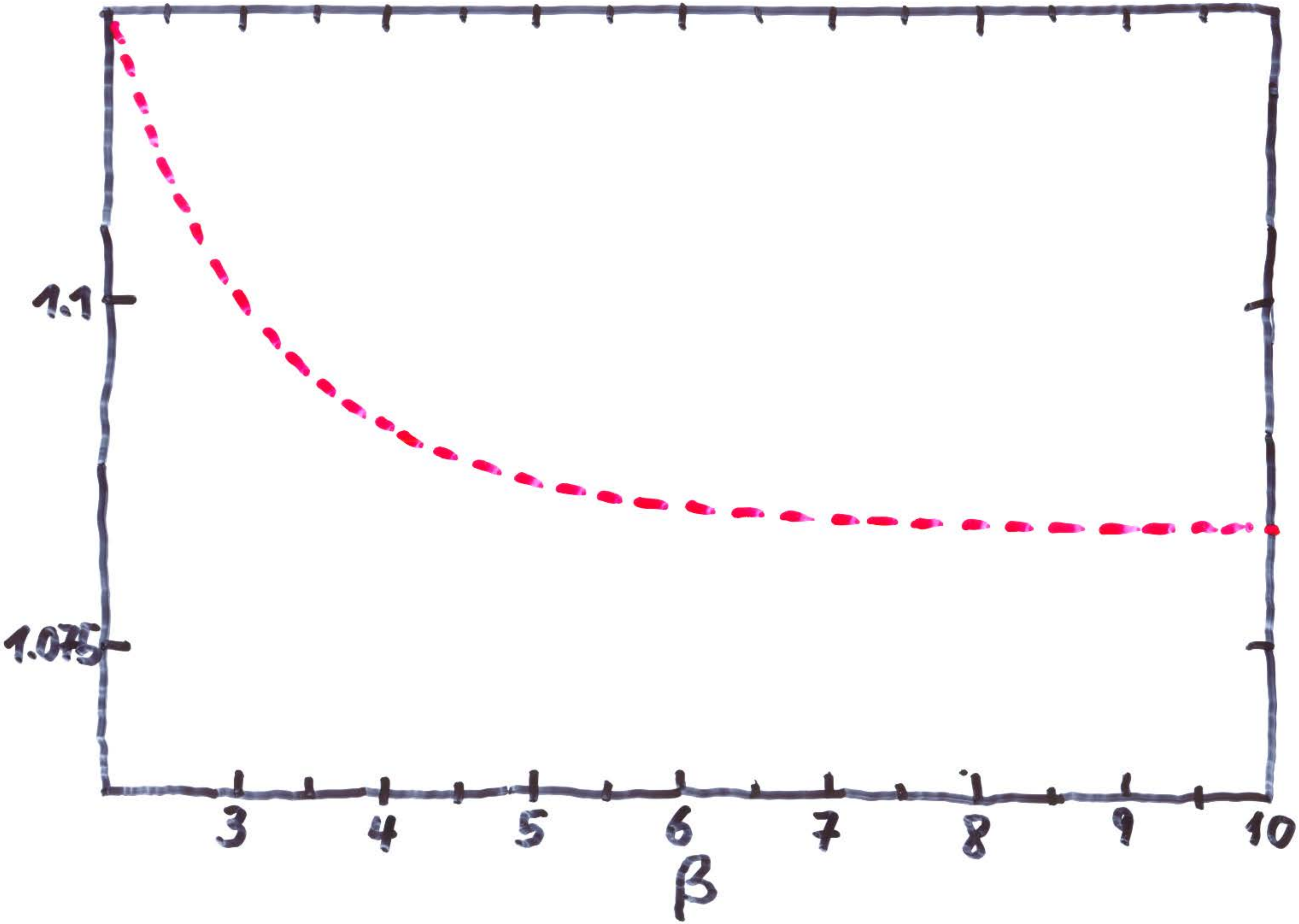
- EXPECTED # CONFIGURATIONS

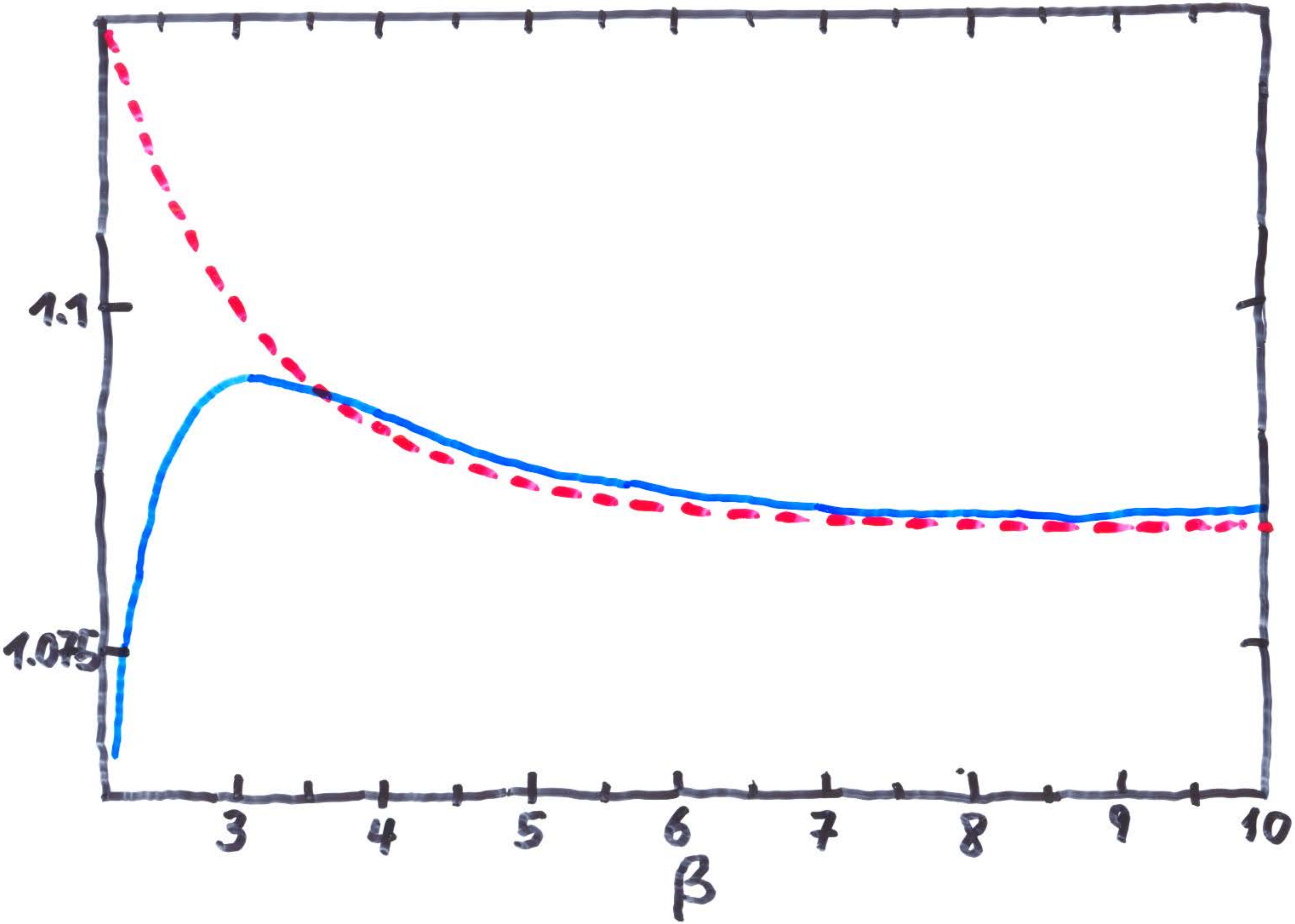


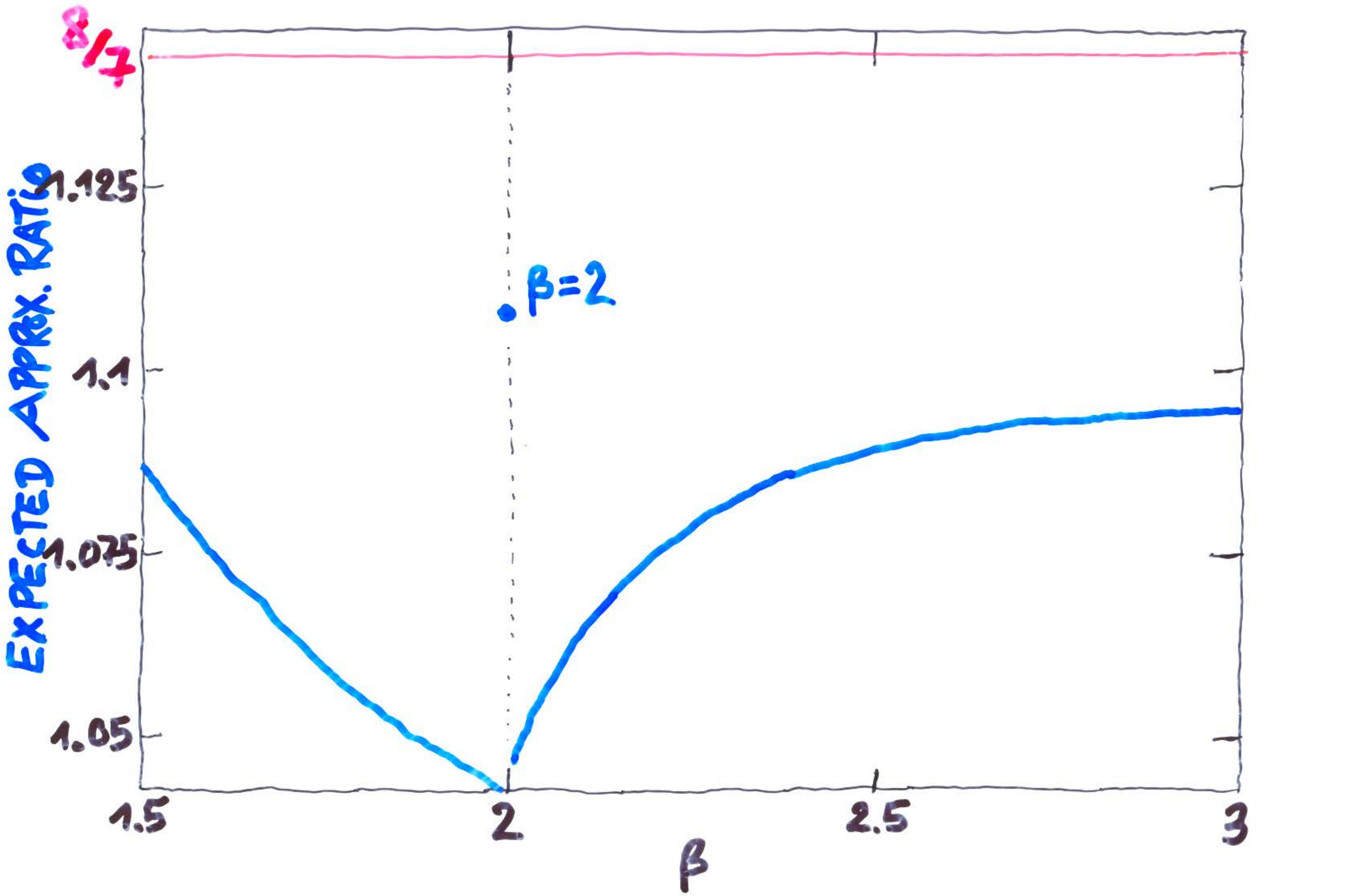
OR



$$\mathbb{E}\left(\#\,\begin{matrix}\text{DEG-1}\\ \text{NEIGHBORS}\end{matrix}\right) \geq \frac{\text{DEG}(v)}{5(\beta-1)} \qquad (\beta > 2)$$


1.1
1.075
3
4
5
6
7
8
9
10
β


1.1
1.075
3
4
5
6
7
8
9
10
β


8/7
1.125
1.1
1.075
1.05
1.5
2
2.5
3
β
EXPECTED APPROX. RATIO
β=2

PLG (1,2)-TSP : LOWER BOUND

PLG (1,2)-TSP : LOWER BOUND

- REDUCTION FROM SUBCUBIC (1,2)-TSP [KS12]

- RESULTING LOWER BOUND:

$$\frac{3^{\beta}\cdot 516\cdot(\mathfrak{S}(\beta)+1)+1}{3^{\beta}\cdot 516\cdot(\mathfrak{S}(\beta)+1)}$$

OUTLINE OF THE CONSTRUCTION

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KARPINSKI, SCHMIED (2012)

HYBRID $\longrightarrow$ SUBCUBIC (1,2)-TSP

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I_H

EQUATIONS

$$x \oplus y \oplus z = 0$$

$$x_j^i \oplus x_k^i = 0$$

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OUR CONSTRUCTION

EQUATIONS

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OUR CONSTRUCTION

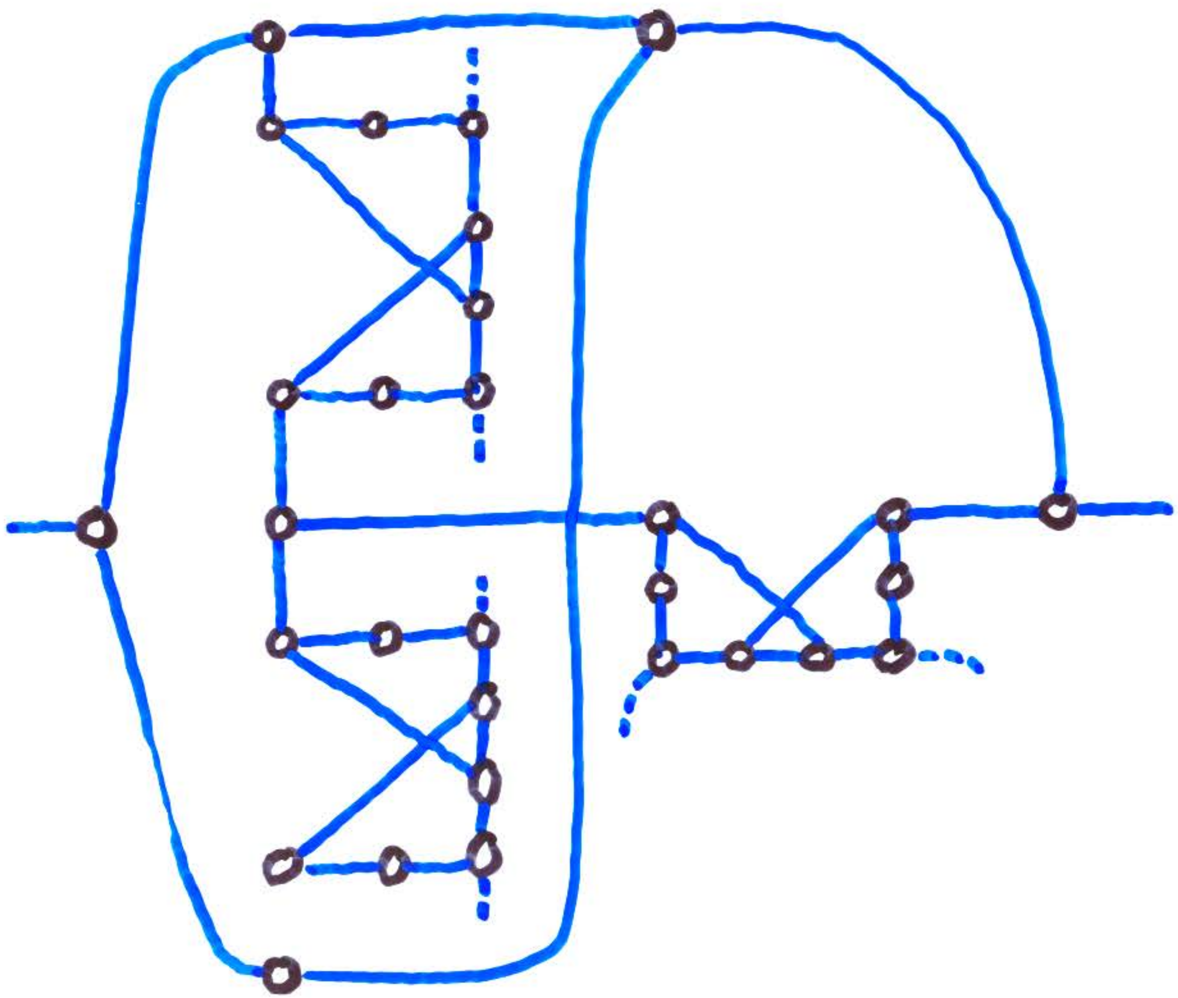
CONTAINS PERFECT MATCHING

EQUATIONS

$$x \oplus y \oplus z = 0$$

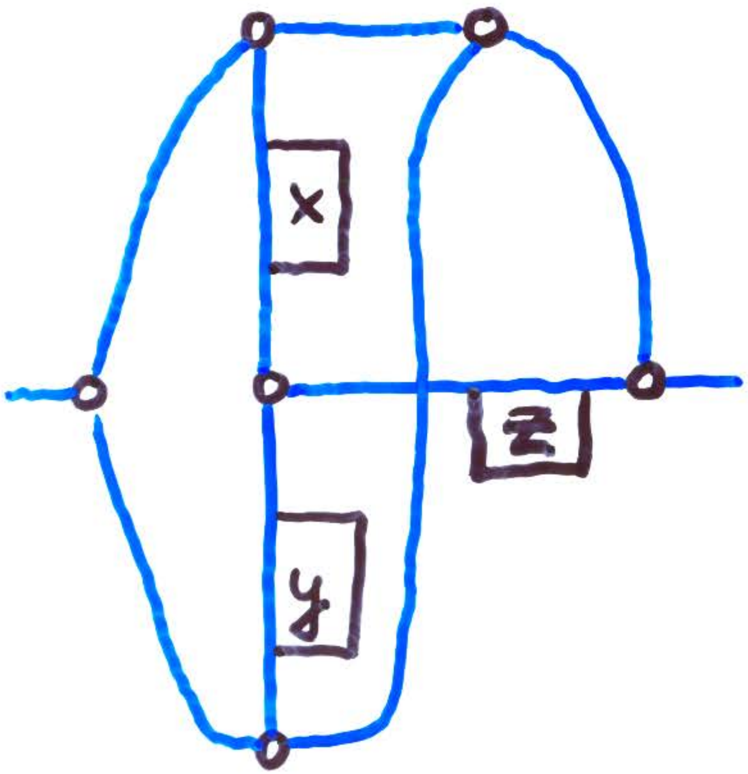
$$x^i_j \oplus x^i_k = 0$$

EQUATION GADGET FOR $x \oplus y \oplus z = 0$



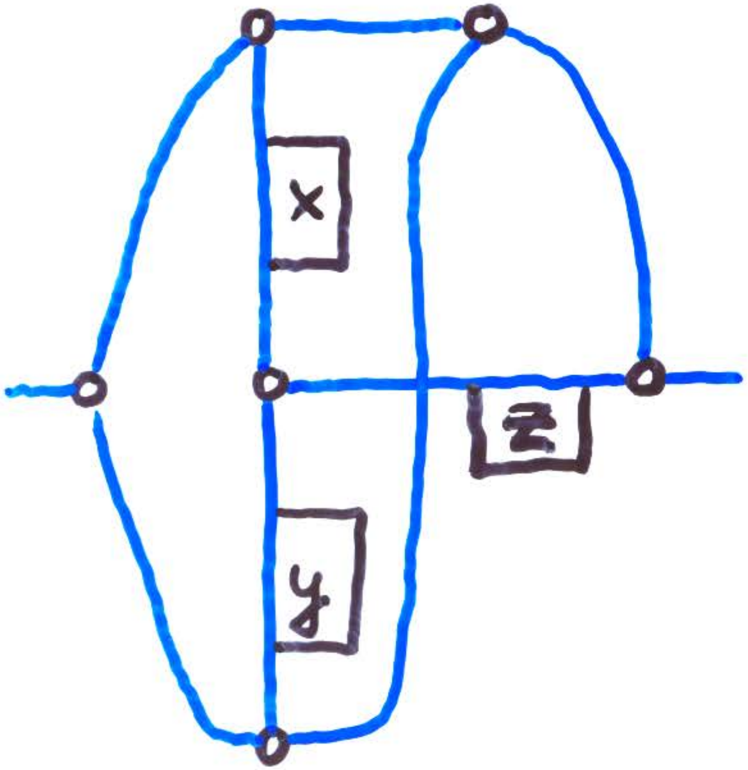
EQUATION GADGET FOR $x \oplus y \oplus z = 0$

MODULAR VIEW

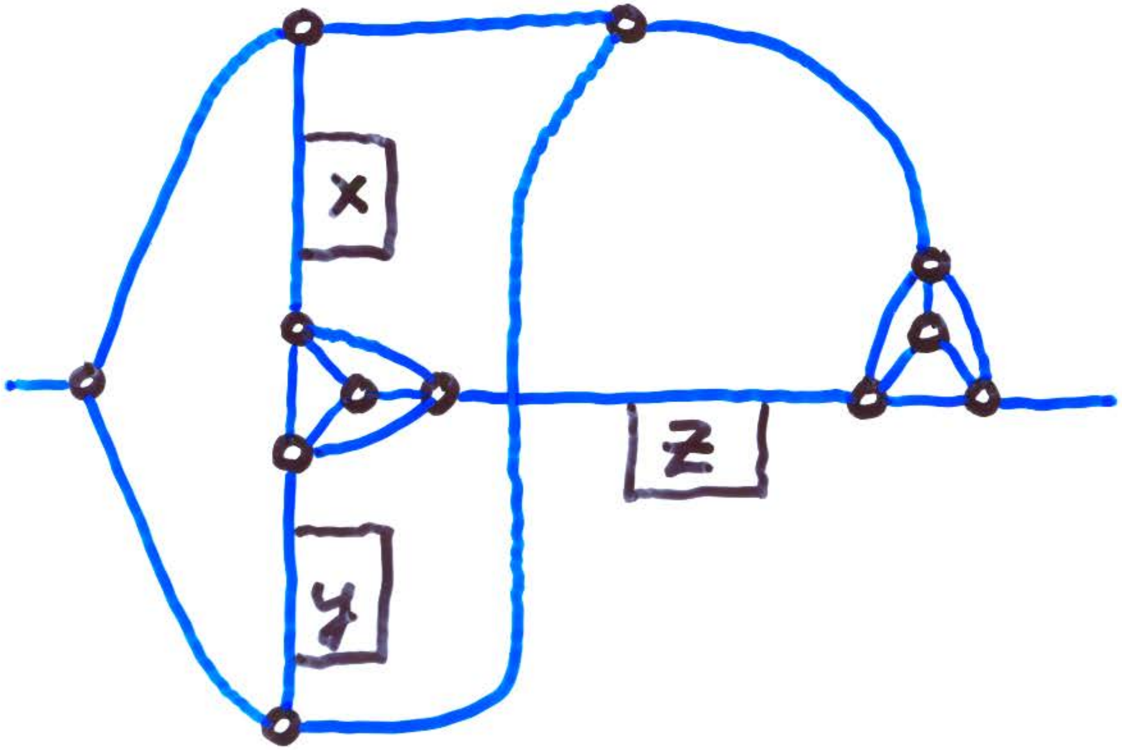


EQUATION GADGET FOR $x \oplus y \oplus z = 0$

MODULAR VIEW

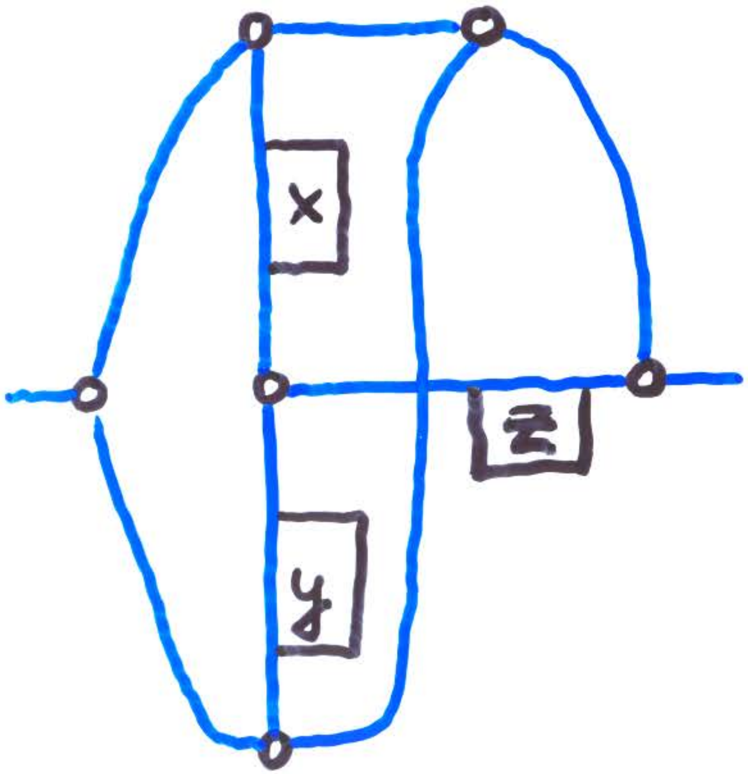


MODIFIED GADGET

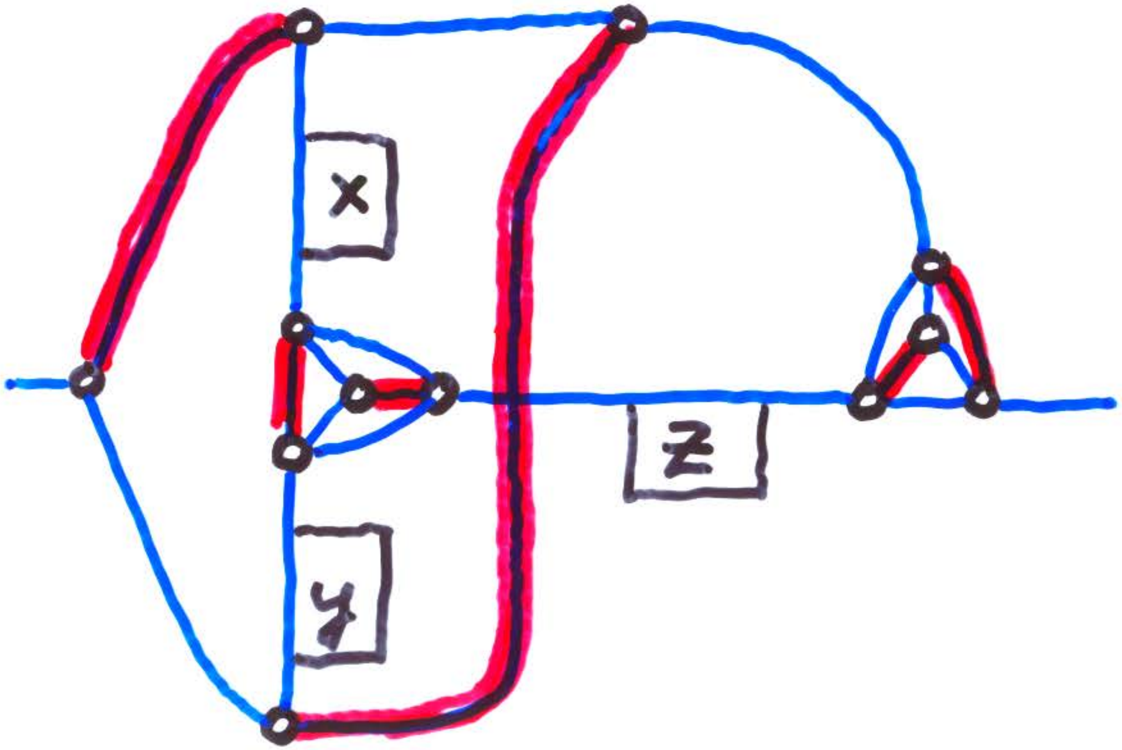


EQUATION GADGET FOR $x \oplus y \oplus z = 0$

MODULAR VIEW



MODIFIED GADGET



CONTAINS PERFECT MATCHING

4 SUMMARY, OPEN PROBLEMS

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- POWER-LAW GRAPHIC TSP FOR $\beta < 2.48$

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? RANDOM POWER LAW GRAPHIC TSP ?

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