A Compendium on Steiner Tree Problems

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Chapter 1

Steiner Tree Problems

1.1 Minimum Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \to \mathbb{R}^+$, set of terminals $S \subseteq V$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$.

COST FUNCTION: $\sum_{e \in E_T} c(e)$

OBJECTIVE: Minimize.

Approx.: Approximable within $\ln 4 + \epsilon < 1.39$ [21] (see also [98],[73]).

Hardness: NP-hard to approximate within an approximation ratio 96/95 [36].

Comment: Admits a PTAS in the special case when G is a planar graph [19]. Solvable exactly in time $O(3^k n + 2^k(n \log n + m))$, where $n = |V|$ is the number of vertices, $k = |S|$ the number of terminals and $m = |E|$ the number of edges in the graph [46],[68].

1.2 Directed Steiner Tree Problem

INSTANCE: Directed graph $G = (V, E)$, edge costs $w : E \to \mathbb{R}^+$, root $r \in V$, set of terminals $S \subseteq V$ of size ($|S| = k$).

SOLUTION: A directed tree $T = (V_T, E_T)$ in G rooted at r such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$.

COST FUNCTION: $\sum_{e \in E_T} w(e)$.

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio $O(|S|^{\epsilon})$ for every $\epsilon > 0$ [27].

Hardness: For every fixed $\epsilon > 0$ cannot be approximated within ratio $\log^{2\text{-}\epsilon} n$, unless $NP \subseteq ZTIME(n^{polylog(n)})$ [61].

Comment: Admits a $O(l^3 t^{2/l})$-approximation algorithm with running time $O(t^{2l} n^l)$, for any integer $l \leq n$, where t is the number of terminals. This gives a $O(t^{\epsilon}/\epsilon^3)$-approximation algorithm with running time $O(t^{4/\epsilon}/n^{2/\epsilon})$ for any fixed $\epsilon > 0$, and an $O(\log^t)$-approximation in quasi-polynomial time [109], [79], [80], [60].

1.3 Steiner Tree Problem with Distances 1 and 2

INSTANCE: Metric space (V, d), set of terminals $S \subset V$, such that for all pairs of vertices $u \neq v$, $d(u, v) \in \{1, 2\}$

SOLUTION: A tree $T = (V_T, E_T)$ such that $S \subseteq V_T \subseteq V$

COST FUNCTION: $d(T) := \sum\limits_{e=\{u,v\} \in E_T} d(u, v)$

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio 1.25 [11].

Hardness: APX- hard [87],[13].

1.4 Metric Steiner Tree Problem

INSTANCE: A finite metric space (V, d) and set of terminals $S \subseteq V$.

SOLUTION: A tree $T = (V_T, E_T)$ such that $S \subseteq V_T \subseteq V$.

COST FUNCTION: $\sum_{e=\{u,v\}\in E_T} d(u, v)$.

OBJECTIVE: Minimize.

Approx.: Approximable within $\ln 4 + \epsilon < 1.39$ [21] (see also [98],[73]).

Hardness: NP-hard to approximate within an approximation ratio 96/95 [36].

Comment: Metric Steiner Tree is equivalent to Minimum Steiner Tree.

1.5 Euclidean Steiner Tree Problem

INSTANCE: Finite set $S \subset \mathbb{R}^2$ of terminals.

SOLUTION: A Steiner tree $T = (V_T, E_T)$ for S with $S \subseteq V_T \subset \mathbb{R}^2$.

COST FUNCTION: The Euclidean length $d_2(T) = \displaystyle\sum_{\{u,v\}\in E_T} ||u - v||_2$ of T, where $|| \cdot ||_2$ denotes the Euclidean Norm in $\mathbb{R}^2$.

OBJECTIVE: Minimize

Approx.: Admits a PTAS [7].

Hardness: NP-hard [7].

Comment: d-dimensional version where $S \subset \mathbb{R}^d$ admits a PTAS for d being constant. For $d = \log(|S|)/\log\log(|S|)$ the problem is APX-hard [102].

1.6 ϵ-Dense Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, set of terminals $S \subseteq V$ such that each $s \in S$ has at least $\epsilon \cdot |V \setminus S|$ neighbors in $V \setminus S$

SOLUTION: A Steiner tree $T = (V_T, E_T)$ for S in G

COST FUNCTION: length $|E_T|$ of T

OBJECTIVE: Minimize

Approx.: For every $\epsilon > 0$, there exists a PTAS for the ϵ-Dense Steiner Tree Problem [74]. This also yields existence of an efficient PTAS [64]). The $\Psi(n)$-Subdense Steiner Tree Problem where every terminal has at least $|V \setminus S|/\Psi(n)$ non-terminal neighbors also admits a PTAS for $\Psi(n) = O(\log(n))$ [25].

Comment: So far the problem is not known to be NP-hard in the exact setting.

1.7 Terminal Steiner Tree

INSTANCE: Graph $G = (V, E)$, cost function $c : E \to \mathbb{R}^+$, set of terminals $S \subset V$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$ and $deg_T(s) = 1$ for every $s \in S$.

COST FUNCTION: $\sum_{e \in E_T} c(e)$.

OBJECTIVE: Minimize.

Hardness: Cannot be approximated within a factor less than $\log^{2-\epsilon} |S|$. This bound also applies to the node-weighted case. [16]

Approx.: Approximable within approximation ratio 2.458 on metric instances [33]. Can be improved to 1.9329 using the algorithm presented in [21].

Comment: For the special case of unit disc graphs, a 20-approximation was given by Biniaz et al. [15]. For the euclidean bottleneck version of this problem, an exact solution can be computed in polynomial time [14].

1.8 Internal Steiner Tree

INSTANCE: Graph $G = (V, E)$, cost function $c : E \to \mathbb{R}^+$, set of terminals $S \subset V$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$ and $deg_T(s) > 1$ for every $s \in S$.

COST FUNCTION: $\sum_{e \in E_T} c(e)$.

OBJECTIVE: Minimize.

Hardness: APX-hard. [69]

Approx.: Approximable within approximation ratio 2ρ on metric instances, where ρ is the the approximation ratio for the Steiner Tree Problem. [107]

Comment: Approximable within approximation ratio $\frac{9}{7}$ on instances where edge weights are restricted to 1 and 2. [70]

1.9 Prize-Collecting Steiner Tree

INSTANCE: Graph $G = (V, E)$, cost function $c : E \to \mathbb{R}^+$, set of terminals $S \subseteq V$, a penalty function $\pi : S \to \mathbb{R}^+$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $V_T \subseteq V$, $E_T \subseteq E$.

COST FUNCTION: $\sum_{e \in E_T} c(e) + \sum_{s \in S \setminus V_T} \pi(s)$.

OBJECTIVE: Minimize.

Approx.: Approximable within $1.9672 - \delta$ for some $\delta > 0$ [6].

Hardness: NP-hard to approximate within 96/95 [36].

Comment: Admits a PTAS for the special case when G is a planar graph [29].

1.10 Bottleneck Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \to \mathbb{R}^+$, set of terminals $S \subseteq V$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$.

COST FUNCTION: $\max_{e \in E_T} c(e)$

OBJECTIVE: Minimize.

Approx.: Can be solved exactly in polynomial time [42][99][41].

1.11 k-Bottleneck Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \to \mathbb{R}^+$, set of terminals $S \subseteq V$, positive integer k.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$ and $|V_T \setminus S| \leq k$ (at most k Steiner nodes used).

COST FUNCTION: $\max_{e \in E_T} c(e)$

OBJECTIVE: Minimize.

Hardness: NP-Hard to approximate within approximation ratio $2 - \epsilon$ on undirected metric graphs [2]. NP-Hard to approximate within approximation ratio $\sqrt{2} - \epsilon$ in the Euclidean plane [103].

Approx.: Approximable within approximation ratio 2 on undirected metric graphs [2]. Approximable within approximation ratio 1.866 in the Euclidean plane [105].

Comment: For the euclidean case, exact algorithms exist for $k = 1$ and $k = 2$, with time complexity $O(n\ log\ n)$ and $O(n^2)$ respectively [8]. For the special case of the euclidean plane with no edges allowed between two Steiner points, a $\sqrt{2} + \epsilon$ approximation algorithm exists [85].

1.12 Prize-Collecting Bottleneck Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \to \mathbb{R}^+$, set of terminals $S \subseteq V$, a penalty function $\pi : S \to \mathbb{R}^+$.

SOLUTION: Tree $T = (V_T, E_T)$ in G such that $V_T \subseteq V$, $E_T \subseteq E$.

COST FUNCTION: $\max(\max_{e \in E_T} c(e), \max_{v \notin V_T} \pi(v))$

OBJECTIVE: Minimize.

Approx.: Can be solved exactly in polynomial time [63].

Comment: This entry covers the penalty-based variant of the Prize-Collecting Steiner Tree Problem. The quota-based Prize-Collecting Steiner Tree Problem, as well as the related Steiner Forest problems can also be solved in polynomial time [63].

1.13 Prize-Collecting k-Bottleneck Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \to \mathbb{R}^+$, set of terminals $S \subseteq V$, positive integer k, a penalty function $\pi : S \to \mathbb{R}^+$.

SOLUTION: Tree $T = (V_T, E_T)$ in G such that $V_T \subseteq V$, $E_T \subseteq E$ and $|V_T \setminus S| \leq k$ (at most k Steiner nodes used).

COST FUNCTION: $\max(\max_{e \in E_T} c(e), \max_{v \notin V_T} \pi(v))$

OBJECTIVE: Minimize.

Hardness: NP-Hard to approximate within $2 - \epsilon$ on undirected metric graphs [2].

Approx.: Approximable within approximation ratio 2 on undirected metric graphs [63].

1.14 Node Weighted Steiner Tree

INSTANCE: Graph $G = (V, E)$, set of terminals $S \subseteq V$ and a node weight function $w : V \to \mathbb{R}^+$.

SOLUTION: A tree $T = (V_T, E_T)$ in G, such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$.

COST FUNCTION: $\sum_{v \in V_T} w(v)$

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio $1.35 \ln k$ [54]. Approximable within approximation ratio $\ln k$ in the unweighted case [54]. The online version admits a polynomial time poly-logarithmic competitive online algorithm [91].

Hardness: NP-Hard [90],[40]. NP-hard to approximate within $(1 - \epsilon)\ln(k)$ for every $\epsilon > 0$ [54].

Comment: Node Weighted Steiner Tree in Unit Disk Graphs is approximable within approximation ratio $(5+\epsilon)$. Admits a PTAS for the special case when the set of vertices is c-local.
A set of vertices S is called c-local in a node weighted graph if in the minimum node weighted spanning tree for S, the weight of longest edge is at most c [84].

1.15 Node Weighted Generalized Steiner Tree

INSTANCE: Graph $G = (V, E)$, node weight function $w : V \to \mathbb{R}^+$, proper function $f\colon P(V) \to \{0, 1\}$

SOLUTION: A subgraph $F = (V_F, E_F)$ of G such that $E_F \cap \delta(S) \geq f(S)$ for all $S \subseteq V$

COST FUNCTION: $\sum_{v \in V_F} w(v)$

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio $1.6103 \ln k$ [54].

Hardness: NP-hard to approximate within $(1 - \epsilon)\ln(k)$ for every $\epsilon > 0$ [54].

1.16 Node Weighted Steiner Network

INSTANCE: Graph $G = (V, E)$, node weights $w\colon V \to \mathbb{R}^+$, edge costs $c\colon E \to \mathbb{R}^+$, k terminal sets $R_1, \ldots, R_k \subseteq V$

SOLUTION: A forest $F = (V_F, E_F)$ in G such that each R_i is contained in a connected component of F

COST FUNCTION: $\sum_{v \in V_F} w(v) + \sum_{e \in E_F} c(e)$

OBJECTIVE: Minimize

Approx.: Approximable within approximation ratio $O(\log k)$ [76]

Hardness: NP-hard to approximate within $(1 - \epsilon)\ln(k)$ for every $\epsilon > 0$ [54].

1.17 Node Weighted Prize Collecting Steiner Tree

INSTANCE: Graph $G = (V, E)$, node weights $w\colon V \to \mathbb{R}^+$, penalties $\pi\colon V \to \mathbb{R}^+$

SOLUTION: A tree $T = (V_T, E_T)$ in G

COST FUNCTION: $\sum_{v \in V_T} w(v) + \sum_{v \in V \setminus V_T} \pi(v)$

OBJECTIVE: Minimize

Approx.: Approximable within approximation ratio $O(\ln |V|)$ [77]

Hardness: NP-hard to approximate within $c \cdot \ln |V|$ for some $c > 0$ [77] For the online version of the problem, there exists an algorithm with polylogarithmic competitive ratio [59].

1.18 Packing Edge-Disjoint Steiner Trees

INSTANCE: An undirected multigraph $G = (V, E)$, set of terminals S $\subseteq$ V.

SOLUTION: A set $\mathcal{T} = \{T_1, ..., T_m\}$ of Steiner trees T_i for S in G which have pairwise disjoint sets of edges.

COST FUNCTION: $|\mathcal{T}|$

OBJECTIVE: Maximize.

Approx.: Approximable within $O(\sqrt{n} \log n)$, where n denotes the number of nodes [34].

Hardness: Not approximable within $(1-\epsilon)\ln(n)$ unless $NP \subseteq DTIME(n^{\log\log n})$. APX-hard for four terminals [34].

1.19 Packing Directed Node-Disjoint Steiner Trees

INSTANCE: Directed multigraph $G = (V, E)$, set of terminals $S \subseteq V$, root $r \in V$.

SOLUTION: A set $\mathcal{T} = \{T_1, ..., T_m\}$ of directed Steiner trees T_i for S rooted at r in G with pairwise disjoint sets of Steiner nodes.

OBJECTIVE: Maximize.

Approx.: Approximable within approximation ratio $O(m^{1/2+\epsilon})$, where m denotes the number of edges[34].

Hardness: NP- hard to approximate within $m^{1/3-\epsilon}$ [34].

1.20 Buy-at-Bulk k-Steiner Tree

INSTANCE: Graph $G(V, E)$, set of terminals $S \subseteq V$, root $s \in S$, an integer $k \leq |S|$, a buy cost function $b : E \rightarrow \mathbb{R}^+$, a distance cost $dist : E \rightarrow \mathbb{R}^+$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$, $s \in V_T$, $|S \cap V_T| \geq k$.

COST FUNCTION: $\sum_{e \in T} b(e) + \sum_{T \in S-s} dist(t, s)$.

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio $O(\log^4 n)$ [58].

Hardness: NP- hard to approximate within $c \cdot \log(n)$ for some constant c [58] .

1.21 Shallow-Light k-Steiner Tree

INSTANCE: Graph $G = (V, E)$, a set of terminals $S \subseteq V$,a buy cost function $b : E \to \mathbb{R}^+$, a distance cost $r : E \to \mathbb{R}^+$, cost bound B and length bound D.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$, $s \in V_T$, $|S \cap V_T| \geq k$, such that the diameter under r-cost is at most D and buy cost is at most B.

COST FUNCTION: $\sum_{e \in E_T} b(e)$

OBJECTIVE: Minimize.

Approx.: Admits an $(O(\log^2 n), O(\log^4 n))$-approximation algorithm [58].

Hardness: NP-hard to approximate within $c \cdot \log(n)$ for some constant c [58].

1.22 General Steiner Tree Star Problem

INSTANCE: Graph $G = (V, E)$, set of terminals $S \subseteq V$, steiner nodes $Y \subseteq V$, the edge weights $c : E \to \mathbb{R}^+$, cost function $w : Y \to \mathbb{R}^+$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$ and $deg_T(v) \leq 1$ for every $v \in V_T \setminus Y$

COST FUNCTION: $\sum_{e \in E_T} c(e) + \sum_{v \in Y \cap V_T} w(v)$.

OBJECTIVE: Minimize .

Approx.: Approximable with 5.16 and 5 [75].

Hardness: Includes the Metric Steiner Tree Problem as a special case, hence it is NP-hard to approximate within an approximation ration 96/95 [36].

Comment: Special case where S and Y are disjoint is called the Steiner Tree Star Problem. This is already NP-hard [75].

1.23 Polymatroid Steiner Problem

INSTANCE: Graph $G = (V, E)$, the edge weights $c : E \to \mathbb{R}^+$, a polymatroid $P = P(V)$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$ and T spans base of P.

COST FUNCTION: $\sum_{e \in E_T} c(e)$

OBJECTIVE: Minimize.

Approx.: Approximable within $O(\log^3 k)$ [24], [27], [32].

Hardness: NP-hard to approximate within $\log^{2-\epsilon} n$ for every $\epsilon > 0$ [61], [24].

Comment: The problem contains the Group Steiner Tree Problem as a special case [24].

1.24 Polymatroid Directed Steiner Problem

INSTANCE: Graph $G = (V, E)$, the edge weights $c : E \to \mathbb{R}^+$, a polymatroid $P = P(V)$.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $S \subseteq V_T \subseteq V$, $E_T \subseteq E$, connecting a given root $r \in V$ to all vertices of a least one base of P.

COST FUNCTION: $\sum_{e \in E_T} c(e)$.

OBJECTIVE: Minimize.

Approx.: Approximable within $O(k^\epsilon)$ for any $\epsilon > 0$, approximable within $O(n^{c \lg n})$ in quasi-polynomial time [24], [27].

Hardness: NP-hard to approximate within $\log^{2\text{-}\epsilon} n$ for every $\epsilon > 0$ [61], [24]

Comment: The problem contains the Directed Steiner Tree Problem as a special case [24].

1.25 Quality of Service Multicast Tree Problem

Instance: Graph $G = (V, E, l, r)$, source $s \in V$, sets of terminals $S_0, ..., S_N$ with node rates $r_0, ..., r_N$ and edge lengths $l : E \to \mathbb{R}^+$.

Solution: A tree $T = (V_T, E_T)$ in G such that $\bigcup_{i \geq 1} S_i \subseteq V_T \subseteq V$, $E_T \subseteq E$.

Cost function: $\sum_{e \in E_T} c(e)$, where $c(e) = l(e) r_e$ (see comment).

Objective: Minimize.

Approx.: Approximable within 1.960 for the case of two non-zero rates. Approximable within 3.802 for the case of unbounded number of rates [72]. For the case of three non-zero rates, the problem admits an 1.522 approximation algorithm [89] [41].

Hardness: NP-hard to approximate within 96/95 [36].

Comment: $r_0 = 0 < r_1 < r_2 < ... < r_N$ are the distinct rates. For $i = 1, \ldots, N$, S_i denotes the set of all nodes with rate r_i. The cost of an edge in the solution tree T is $c(e) = l(e) r_e$, where r_e (rate of edge e)is the maximum rate in the component $T - e$

1.26 Zero Skew Tree Problem

Instance: Metric space (M, d), set of sinks $S \subseteq M$, edge costs $cost : E \to \mathbb{R}^+$.

SOLUTION: A stretched tree $T = (V_T, E_T, \pi, c)$ consisting of an arborescence (V_T, E_T), a mapping $\pi : V_T \to M$ such that π is a one-to-one mapping between the leaves of T and S and a cost function $c\colon E_T \to \mathbb{R}_+$ such that for every edge (u, v) of T, $c(u, v) \geq d(\pi(u), \pi(v))$ and furthermore, for each pair P, P' of root-to-leaf paths in T, $c(P) = c(P')$.

COST FUNCTION: $\sum_{(u,v)\in E_T} c(u, v)$

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio 4 when the root is not fixed as a part of the instance [110]. Approximable within approximation ratio $2e$ if the root is fixed [28].

Hardness: NP-hard [28].

Comment: The complexity of the rectilinear zero skew tree problem is not known. For a fixed tree topology, the problem can be solved in linear time by using the Deferred-Merge Embedding (DME) [17], [26], [43].

1.27 Bounded Skew Tree Problem

INSTANCE: Metric space (M, d), set of sinks $S \subseteq M$, edge costs $cost$: $E \to \mathbb{R}^+$, bound$b$.

SOLUTION: A stretched tree $T = (V_T, E_T, \pi, c)$ consisting of an arborescence (V_T, E_T), a mapping $\pi : V_T \to M$ such that π is a one-to-one mapping between the leaves of T and S and a cost function $c\colon E_T \to \mathbb{R}_+$ such that for every edge (u, v) of T, $c(u, v) \geq d(\pi(u), \pi(v))$ and furthermore, for each pair P, P' of root-to-leaf paths in T, $|c(P) - c(P')| \leq b$

COST FUNCTION: $\sum_{(u,v)\in E} cost(u, v)$

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio 14 when the root is not fixed as a part of the instance [110]. Approximable within 16.86 when the root is given as part of the input [28].

Hardness: NP-hard [28]. Also NP-hard in the two-dimensional rectilinear case [110].

1.28 Stochastic Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, the root r, first stage edge costs $c_e \geq 0, e \in E$, inflation parameter σ and probability distribution π on the set of scenarios K, where a scenario k is a set of terminal nodes.

SOLUTION: A subset of edges $E_0 \subseteq E$ to be purchased in the first stage

COST FUNCTION: $\sum_{e \in E_0} c_e + \mathbb{E}[\sum_{e \in E_k} c_e \sigma]$ while $E_o \cup E_k$ spans S_k for every $k \in K$, where E_k is the set of edges that have to be bought in the second stage to connect all terminal nodes.

OBJECTIVE: Minimize.

Approx.: Approximable within 3.39 [101], [56].

Hardness: NP-hard.

Comment: In this model there are two separate stages: First stage, where G, r, c_e, σ and π are known. In this stage one must purchase set of edges E_0 that is predicted to be useful for connecting unknown set of vertices k that will be drawn from K according to π. Second stage, where k is revealed, cost of every edge increases by a factor of σ and a set of edges E_k has to be bought to connect all terminals. [49], [18].

1.29 Group Steiner Tree Problem

INSTANCE: Graph $G = (V, E)$, edge cost $c : E \to \mathbb{R}^+$, and sets $S_1, ..., S_n \subseteq V$, also called groups.

SOLUTION: A tree $T = (V_T, E_T)$ in G which contains at least one terminal from every group S_i

COST FUNCTION: $\sum_{e \in E_T} c(e)$.

OBJECTIVE: Minimize.

Approx.: Approximable within $O(\log^3 n)$. Approximable within $O(\log^2 n)$ if G is a tree [53], [40].

Hardness: Not approximable within $\Omega(\log^{2\text{-}\epsilon} n)$ unless NP admits quasipolynomial-time Las Vegas algorithm [61].

Comment: Approximable within $O(\log n polylog \log n)$ when the graph is planar and each group is the set of nodes on a face [40].

1.30 Two-Level Rectilinear Steiner Tree

INSTANCE: Set of terminals $S \subset \mathbb{R}^2$ in the plane, partition of S into k subsets $S_1, \ldots, S_k$.

SOLUTION: Two-Level rectilinear Steiner Tree T for S in $\mathbb{R}^2$ consisting of Steiner Trees T_i for S_i $(i = 1, \ldots, k)$ and a top-level tree T_0 connecting the trees $T_1, \ldots, T_k$

COST FUNCTION: $c(T_0) + \sum_{i=1}^{k} c(T_i)$, where $c(\,)$ denotes the L_1-length of the trees

OBJECTIVE: Minimize.

Approx.: Approximable within 1.63, based on the PTAS for Rectilinear Steiner Tree in the plane [66].

Comment: Admits a PTAS for the case when k is fixed [66].

1.31 Fractional Steiner Tree Problem with Profits

INSTANCE: Graph $G = (V, E)$, a cost function $c : E \to \mathbb{R}^+$, a revenue function $r : V \to \mathbb{R}^+$, fixed cost $c_0 \geq 0$

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $V_T \subseteq V$, $E_T \subseteq E$.

COST FUNCTION: $\dfrac{\displaystyle\sum_{v \in V_T} r(v)}{c_0 + \displaystyle\sum_{e \in E_T} c(e)}$.

OBJECTIVE: Maximize.

Hardness: NP-hard [39].

1.32 Budget Steiner Tree Problem with Profits

INSTANCE: Graph $G = (V, E)$, a cost function $c : E \to \mathbb{R}^+$, a revenue function $r : V \to \mathbb{R}^+$, budget B.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $V_T \subseteq V$, $E_T \subseteq E$ and $\sum_{e \in E_T} c(e) \leq B$.

COST FUNCTION: $\sum_{v \in V_T} r(v)$

OBJECTIVE: Maximize.

Approx.: Approximable within $(4 + \epsilon)$ for every $\epsilon > 0$ [83]

Hardness: NP-hard.

1.33 Quota Steiner Tree Problem with Profit

INSTANCE: Graph $G = (V, E)$, a cost function $c : E \to \mathbb{R}^+$, a revenue function $r : V \to \mathbb{R}^+$, quota Q.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $V_T \subseteq V$, $E_T \subseteq E$ and $\sum_{v \in V_T} r(v) \geq Q$.

COST FUNCTION: $\sum_{e \in E_T} c(e)$.

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio 2.5 [71].

Hardness: NP-hard to approximate within 96/95 [36].

Comment: The lower bound follows from the fact the the Quota Steiner Tree Problem with Profit contains the Steiner Tree Problem as a special case.

1.34 Steiner Tree Problem in Phylogeny

INSTANCE: $C = 1, \cdots m$ set of Characters, for each $i \in C$, set of states A_i of character i, $S \subseteq A_1 \times \cdots \times A_m$ set of species.

SOLUTION: A tree $T = (V_T, E_T)$ with $S \subseteq V_T \subseteq A_1 \times \cdots \times A_m$

COST FUNCTION: $\sum_{e=u,v \in E_T} d_H(u, v)$, where d_H denotes the Hamming distance [48].

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio $\ln(4) + \epsilon$ for every $\epsilon > 0$ [4],[65].

Hardness: NP-hard [48].

Comment: A phylogeny for a set of n distinct species S is a tree whose leaves are all elements of S and where $S \subseteq V(T) \subseteq A_1 \times \cdots \times A_m$.

1.35 Steiner Tree Problem in Phylogeny with Given Topology

INSTANCE: Set of characters $C = 1, \cdots m$, for each $i \in C$ a set A_i of states of character i, $\lambda\colon S \to L(T)$ between S and the set $L(T)$ of leaves of T

SOLUTION: An assignment $a\colon V(T) \setminus L(T) \to A_1 \times \cdots \times A_m$.

COST FUNCTION: $\sum_{e=u,v \in E_T} d_H(a(u), a(v))$, where d_H denotes the Hamming distance

OBJECTIVE: Minimize.

Approx.: Admits a PTAS [104].

Comment: This is a special case of the Tree Alignment with a Given Phylogeny.

1.36 Tree Alignment with a Given Phylogeny

INSTANCE: Finite set of strings $S \subset \Sigma^*$ over a given finite alphabet, arborescence T, bijection $\lambda\colon S \to L(T)$ between S and the set $L(T)$ of leaves of T, scoring scheme $\mu\colon (\Sigma \cup \{-\})^2 \to \mathbb{R}$

SOLUTION: An assignment $a\colon V(T) \setminus L(T) \to (\Sigma \cup \{-\})^*$ such that for each internal node u of T, $a(u)$ is an alignment of the strings $a(v)$ assigned to all the children v of u in T

COST FUNCTION: $\displaystyle\sum_{e=u,v \in E_T} \mu(a(u), a(v))$

OBJECTIVE: Minimize.

Approx.: Admits a PTAS when the cost function given by the scoring scheme μ is a metric [104].

Hardness: NP-hard. In the case of general scoring schemes, the problem becomes APX-hard [104].

Comment: Augmenting the construction with a local optimization technique, for each $t > 1$, has a performance ratio $1 + 3/t$ [104].

1.37 Minimum-Cost 2-Edge-Connected Augmentation of Tree with Constant Radius

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \to \mathbb{R}^+$, a tree F on V disjoint to E.

SOLUTION: A tree $T = (V_T, E_T)$ in G such that $V_T \subseteq V$, $E_T \subseteq E$, and $T \cup F$ is 2-edge-connected.

COST FUNCTION: $\sum_{e \in E_T} c(e)$

OBJECTIVE: Minimize.

Approx.: Approximable within $1 + ln(2)$ [38].

Hardness: NP-hard to approximate for trees with radius ≥ 2 [50].

1.38 Minimum-Cost (k, p)-Steiner Tree with Limited Number of Branching Nodes

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \to \mathbb{R}^+$, a set $S \subset V$ of k terminals.

SOLUTION: A Steiner tree $T = (V_T, E_T)$ for S in G such that T contains at most p branching nodes.

COST FUNCTION: $\sum_{e \in E_T} c(e)$

OBJECTIVE: Minimize.

Approx.: For p being constant, solvable in polynomial time when the input graph is acyclic or when k is also fixed and the input graph is of bounded treewidth [106]

Hardness: NP-hard to approximate within $n^{1-\epsilon}$ for every $\epsilon > 0$ when k is not fixed, even in planar graphs with unit edge costs [106]

1.39 Minimum-Cost (k, p)-Directed Steiner Tree with Limited Number of Branching Nodes

INSTANCE: Directed Graph $G = (V, E)$, edge costs $w : E \to \mathbb{R}^+$, root $r \in V$, set of terminals $S \subset V$ of size $k \geq 2$.

SOLUTION: A directed tree $T = (V_T, E_T)$ in G rooted at r such that $S \subseteq V_T \subseteq V$ and T contains at most p branching nodes

COST FUNCTION: $\sum_{e \in E_T} c(e)$

OBJECTIVE: Minimize.

Hardness: NP-hard to approximate within $n^{1-\epsilon}$ for every $\epsilon > 0$ when k is not fixed, even for planar graphs with unit edge costs [106]

Comment: When both k and p are fixed, deciding existence of a feasible solution is in P [106].

1.40 Packing Element-Disjoint Steiner Trees on Bounded Terminal Nodes

INSTANCE: Graph $G = (V, E)$, set of terminals $S \subseteq V$.

SOLUTION: A set $\mathcal{T} = \{T_1, ..., T_m\}$ of Steiner trees T_i for S in G with pairwise disjoint sets of Steiner nodes.

COST FUNCTION: m (the number of trees)

OBJECTIVE: Maximize.

Approx.: Approximable within $\lceil \frac{|S|}{2} \rceil$ [67].

Hardness: APX-hard even for $|S| = 3$ [1]. NP-hard to approximate within $O(log|V|)$ [34].

Chapter 2

Steiner Forest Problems

2.1 Steiner Forest Problem

Instance: Graph $G = (V, E)$, cost function $c : E \to \mathbb{R}_+$, set of k terminal pairs $S = \{(s_1, t_1), ..., (s_k, t_k)\}$.

Solution: A forest $F \subseteq E$ such that for all $1 \leq j \leq k$, vertices s_j and t_j are contained in the same connected component of F

Cost function: $\sum_{e \in F} c_e$.

Objective: Minimize.

Approx.: Approximable within $2 - 1/k$ [3].

Hardness: NP-hard to approximate within 96/95 [36].

Comment: When G is a planar graph [9] obtained a PTAS.

2.2 k-Steiner Forest Problem

Instance: Graph $G = (V, E)$, cost function $c : E \to \mathbb{R}_+$, set of demands $D = \{(s_1, t_1), ..., (s_l, t_l)\}$, integer $k \leq l$

Solution: A forest F in G such that at least k pairs from D are connected by F.

Cost function: $\sum_{e \in F} c_e$.

OBJECTIVE: Minimize.

Approx.: Approximable within $O(min\{n^{2/3}, \sqrt{l}\} \log k)$ [111].

Hardness: NP-hard.

2.3 Steiner Forest with Distances One and Two

INSTANCE: Graph $G = (V, E)$, and a collection $\mathcal{R}$ of subsets $R \subseteq V$ called required sets, where $\bigcup_{R \in \mathcal{R}} R$ is the set of *terminals*.

SOLUTION: A set of unordered node pairs F such that each $R \in \mathcal{R}$ is contained in a connected component of (V, F).

COST FUNCTION: $|F \cap E| + 2|F - E|$.

OBJECTIVE: Minimize.

Approx.: Approximable within 3/2 [12].

Hardness: APX- hard.[87],[13]

Comment: G defines a $\{1, 2\}$-Metric on V where E is the set of node pairs which are at distance one from each other, and all other node pairs are at distance 2

2.4 Degree Bounded Survivable Network Design

INSTANCE: Graph $G = (V, E)$, edge costs $c : E \rightarrow \mathbb{R}_+$, degree bounds $b_v, v \in V$, requirements $r_{uv}, u, v \in V$

SOLUTION: A subgraph H of G which contains for each pair of vertices u, v at least r_{uv} edge-disjoint paths from u to v and such that for all $v \in V$, $d_H(v) \leq b_v$

COST FUNCTION: $c(H)$

OBJECTIVE: Minimize.

Approx.: There exists an algorithm which constructs a subgraph H of cost at most 2 times the optimum cost such that H satisfies all the connectivity requirements r_{uv} and such that $d_H(v) \leq \min\{b_v + 3r_{\max}, 2b_v + 2\}$, where $r_{\max} = \max_{u,v} r_{uv}$ [82].

Comment: There exists a $(O(1), O(1))$ bicriteria approximation algorithm for Degree Bounded Survivable Network Design with element-connectivity requirements, where the paths satisfying the connectivity requirements have to be element disjoint [45]. For the Degree Bounded Survivable Network Design problem with node-connectivity requirements, there exists a $(O(k^3 \log n), O(k^3 \log n))$ bicriteria approximation algorithm, where k is the maximum connectivity requirement of any pair [45].

2.5 Strongly Connected Steiner Subgraph

INSTANCE: A directed graph $G = (V, E)$, edge weights $c_e, e \in E$, set of terminals $S = \{s_1, \ldots, s_p\}$

SOLUTION: A set of edges $H \subset E$ such that for all $1 \leq i, j \leq p, i \neq j$ the induced subgraph $G[H]$ contains a directed s_i, s_j-path

COST FUNCTION: $c(H)$

OBJECTIVE: Minimize.

Approx.: Approximable within p^ϵ for every $\epsilon > 0$ [27].

Hardness: For every fixed $\epsilon > 0$, the SCSS cannot be approximated within ratio $\log^{2-\epsilon} n$, unless $NP \subseteq ZTIME(n^{polylog(n)})$ [61].

Comment: In a variant 2-SCSS(k_1, k_2), the number of terminals is $p = 2$, and the task is to construct a subset $H \subseteq E$ such that $G[H]$ contains k_1 s_1, s_2-paths and k_2 s_2, s_1-paths. The objective is to minimize $\sum_{e \in H} c_e \cdot \phi(e)$, where $\phi(e)$ is the maximum number of s_1, s_2-paths or s_2, s_1-paths using edge e. The 2-SCSS$(k, 1)$ can be solved in $n^{O(k)}$ time but does not have an $f(k) \cdot n^{o(k)}$ algorithm for any computable function f, unless the Exponential Time Hypothesis (ETH) fails [35].

2.6 Directed Steiner Forest (DSF)

INSTANCE: A directed graph $G = (V, E)$, an edge cost function $c : E \to \mathbb{R}_+$, a collection $D \subseteq V \times V$ of ordered node pairs, and an integer $k = |D|$.

SOLUTION: A subgraph F of G that containes an shortest path for (at least) k pairs $(s, t) \in D$.

COST FUNCTION: $\sum_{e \in F} c_e$.

OBJECTIVE: Mininimize.

Approx.: Approximable within $O(n^{\frac{2}{3}+\epsilon})$ for every $\epsilon > 0$ [10].

Hardness: NP-hard to approximate within $\log(n)^{2-\epsilon}$.

Comment: The k-Directed Steiner Forest (k-DSF) is approximable within $O(k^{1/2+\epsilon})$ for every $\epsilon > 0$ [47].

2.7 Prize-Collecting Steiner Forest

INSTANCE: Graph $G = (V, E)$, set of terminal pairs $S = \{(s_i, t_i)\}_{1 \le i \le k}$, cost function $c : E \to \mathbb{R}_+$, penalty function $\pi : S \to \mathbb{R}_+$.

SOLUTION: A pair (F, Q), where F is a forest and $Q \subseteq S$ contains all the terminal pairs that are not connected by F

COST FUNCTION: $c(F) + \pi(Q)$.

OBJECTIVE: Minimize.

Approx.: Approximable within approximation ratio 3 using a primal-dual approach. Approximable within approximation ratio 2.54 by an LP-Rounding algorithm [100],[57].

Hardness: NP-hard to approximate within 96/95 [36].

Comment: PTAS exists for the special case when G is planar graph [29].

2.8 Prize-Collecting Node Weighted Survivable Network Design

INSTANCE: Graph $G = (V, E)$, connectivity requirements $r(u, v) \in \mathbb{Z}_{\geq 0}$ and penalties $\pi(u, v) \geq 0$ for all $u, v \in V$, node weights $w \colon V \to \mathbb{R}_+$

SOLUTION: Subgraph H in G

COST FUNCTION: weight $w(H)$ of H plus the sum of penalties $\pi(u, v)$ for which H does not contain at least $r(u, v)$ edge-disjoint $u - v$ paths

OBJECTIVE: Minimize

Approx.: Approximable within approximation ratio $O(k^2 \log n)$, where $k = \max_{u,v \in V} r(u, v)$ [31]. Approximable within approximation ratio $O(k)$ for minor-closed families of graphs [30].

2.9 Packing Steiner Forest

INSTANCE: Undirected multigraph $G = (V, E)$, set $\mathcal{S} = \{S_1, ..., S_t\}$ of pairwise disjoint subsets S_i of V

SOLUTION: A set $\mathcal{F}$ of pairwise edge-disjoint Steiner forests F_i for $\mathcal{S}$ in G

COST FUNCTION: $|\mathcal{F}|$ (the number of Steiner forests)

OBJECTIVE: Maximize.

Approx.: APX [81].

Hardness: NP-hard.

Comment: If each S_i is Qk-edge-connected in G, then there are k edge-disjoint S-forests in G. The best upper bound achieved on Q is 32. This yields the first polynomial time constant factor approximation algorithm for the Steiner Forest Packing problem. [81]

2.10 Stochastic Steiner Forest

INSTANCE: A graph $G = (V, E)$, an edge cost function $c : E \to \mathbb{R}_+$, a probability distribution π over sets of source-sink pairs $2^{\binom{V}{2}}$, and an *inflation* parameter $\lambda \geq 1$.

SOLUTION: A set of first-stage edges E_0 and for each $D \in 2^{\binom{V}{2}}$, a set of second-stage edges E_D such that (i) the edges in $E_0 \bigcup E_D$ connect each of the pairs in D.

COST FUNCTION: $\sum_{e \in E_0} c_e + \mathbb{E}_{D \leftarrow \pi}[\sum_{e \in E_0} c_e]$.

OBJECTIVE: Minimize.

Approx.: Approximable within 5 [55][49].

Hardness: NP hard.

Comment: A basic building block is an s-star consisting of a non-terminal c, called the center, s terminals $t_1, ..., t_s$ and edges $(c, t_1), ..., (c, t_s)$.

2.11 Steiner Activation Network

INSTANCE: Graph $G = (V, E)$, monotone activation function $f_e : \mathbb{R}^+ \times \mathbb{R}^+ \to \{0, 1\}$ for each edge $e \in E$, terminal sets $R_1, \ldots, R_k \subseteq V$

SOLUTION: Assignment $x = (x_v)_{v \in V} \in \mathbb{R}^{|V|}$ such that the subgraph induced by the activated edges $e = \{u, v\}$ (i.e. $f_e(x_u, x_v) = 1$) connects every terminal set R_i

COST FUNCTION: $\sum_{v \in V} x_v$

OBJECTIVE: Minimize

Approx.: Approximable within approximation ratio $O(\log k)$ [93]

Hardness: NP-hard to approximate within $o(\log k)$ [93]

2.12 Bifamily Edge Cover Activation

Instance: Graph $G = (V, E)$, monotone activation functions $f_e \colon \mathbb{R}^+ \times \mathbb{R}^+ \to \{0, 1\}$ for the edges $e \in E$, bifamily $\mathcal{F}$ of subsets of V

Solution: assignment $x = (x_v)_{v \in V} \in \mathbb{R}^{|V|}$ such that the set E_x of activated edges $e = \{u, v\}$ (i.e. $f_e(x_u, x_v) = 1$) covers $\mathcal{F}$

Cost function: $\sum_{v \in V} x_v$

Objective: Minimize

Approx.: Admits an $O(\log |\mathcal{C}_{\mathcal{F}}|)$-approximation algorithm for the case when $\mathcal{F}$ is an uncrossable family, where $\mathcal{C}_{\mathcal{F}}$ is the set of all subsets X for which $\hat{X} \in \mathcal{F}$ and X does not contain two distinct inclusion-minimal members of the family $\{X | \hat{X} \in \mathcal{F}\}$ [92]

Comment: A bifamily $\mathcal{F}$ of subsets of V is a set of pairs $\hat{X} = (X, X^+)$ of subsets of V such that for each $\hat{X} = (X, X^+)$, $X \subseteq X^+$ and the following property holds: For all $\hat{X} = (X, X^+)$ and $\hat{Y} = (Y, Y^+)$ in $\mathcal{F}$, $X = Y$ implies $X^+ = Y^+$ and $X \subseteq Y$ implies $X^+ \subseteq Y^+$. A set of edges $E' \subseteq E$ covers $\mathcal{F}$ if for each $\hat{X} = (X, X^+)$ in $\mathcal{F}$, there is an edge $e \in E'$ which goes from $V \setminus X^+$ to X.

2.13 Network Activation with Property Π

Instance: Graph $G = (V, E)$, monotone activation functions $f_e \colon \mathbb{R}^+ \times \mathbb{R}^+ \to \{0, 1\}$ for the edges $e \in E$, monotone property Π of subgraphs of G

Solution: Assignment $x = (x_v)_{v \in V} \in \mathbb{R}^{|V|}$ such that the subgraph induced by the activated edges $e = \{u, v\}$ (i.e. $f_e(x_u, x_v) = 1$) is contained in Π

Cost function: $\sum_{v \in V} x_v$

Objective: Minimize

Approx.: If every inclusion minimal edge-set $F \subseteq E$ with $(V, F) \in \Pi$ has maximum degree at most Δ and the underlying Edge-Costs

Network Design Problem with property Π admits a θ-approximation algorithm, the problem is approximable within approximation ratio $\theta\Delta$ [92, 51].

2.14 Euclidean Steiner Forest Problem

INSTANCE: Finite set of k terminal pairs $S = \{(s_1, t_1), ..., (s_k, t_k)\} \subset \mathbb{R}^2$.

SOLUTION: A forest F such that for all $1 \leq j \leq k$, vertices s_j and t_j are contained in the same connected component of $F, F \subset \mathbb{R}^2$.

COST FUNCTION: The Euclidean length $d_2(F) = \sum_{\{u,v\} \in F} ||u - v||_2$ of F, where $|| \cdot ||_2$ denotes the Euclidean Norm in $\mathbb{R}^2$.

OBJECTIVE: Minimize

Approx.: Admits a PTAS [20].

Hardness: NP-hard [7].

Comment: d-dimensional version where $S \subset \mathbb{R}^d$ admits a PTAS for d being constant. For $d = \log(|S|)/\log\log(|S|)$ the problem is APX-hard [102].

Chapter 3

Broadcast

3.1 Minimum Broadcast Time

INSTANCE: Graph $G = (V, E)$ and a source node $v_0 \in V$.

SOLUTION: A broadcasting scheme. At time 0 only v^0 contains the message that is to be broadcast to every vertex. At each time step any vertex that has received the message is allowed to communicate the message to at most one of its neighbours.

COST FUNCTION: The broadcast time, i.e., the time when all vertices have received the message.

OBJECTIVE: Minimize.

Approx.: Approximable within $O(\log^2 |V| / \log \log |V|)$ [96].

Hardness: NP-hard [52].

Comment: Approximable within $2B$ if the degree of G is bounded by a constant B [96]. Approximable within $O(\log V)$ if G is chordal, k-outerplanar [78]. Approximable within $O(\log |V| / \log \log |V|)$ if G has bounded tree width [88].

3.2 Minimum-Energy Broadcast Tree Problem

Instance: Wireless ad-hoc network $M = (N, L)$ consisting of set of nodes N, location function $L : N \to \mathbb{Z}_+^2$, for each node $v_i \in N, k$ power levels $w_{i,1} \leq w_{i,2} \leq ... \leq w_{i,k}$, a receiver sensitivity $\vartheta > 0$ propagation function $\gamma\colon \mathbb{Z}^2 \times \mathbb{Z}^2 \to \mathbb{R}_+$

Solution: An arborescence $T = (N, E)$, power assignment $p = (p_i)_{v_i \in N}$ with $p_i \in \{w_{i,1}, \ldots, w_{i,k}\}$ for all $v_i \in N$ such that for each directed edge $e = (v_i, v_j)$ in T, $p_i - \gamma(L(v_i), L(v_j)) \geq \vartheta$

Cost function: $\sum_{v_i \text{ non-leaf in } T} p_i$

Objective: Minimize.

Approx.: Approximable within $O((k+1)^{1/\epsilon} n^{3/\epsilon}$, where $n = |N|$ is the number of nodes in the wireless network, k is the number of power levels at each node, and ϵ is constant with $0 < \epsilon < 1$ [86].

Hardness: NP-hard [86].

Comment: When every node is equipped with the same type of battery, an approximation algorithm has a better performance ratio than that in the general case setting, and the algorithm takes $O(kn^2 \log n)$time [86].

3.3 Minimum-Energy Multicast Tree Problem

Instance: Wireless ad-hoc network $M = (N, L)$ consisting of set of nodes N, location function $L : N \to \mathbb{Z}_+^2$, for each node $v_i \in N, k$ power levels $w_{i,1} \leq w_{i,2} \leq ... \leq w_{i,k}$, a receiver sensitivity $\vartheta > 0$ propagation function $\gamma\colon \mathbb{Z}^2 \times \mathbb{Z}^2 \to \mathbb{R}_+$, set of destinations $D \subseteq N$, a source $s \in N \setminus D$

Solution: An arborescence $T = (V, E)$ rooted at s with $D \subseteq V \subseteq N$, power assignment $p = (p_i)_{v_i \in V}$ with $p_i \in \{w_{i,1}, \ldots, w_{i,k}\}$ for all $v_i \in V$

such that for each directed edge $e = (v_i, v_j)$ in T, $p_i - \gamma(L(v_i), L(v_j)) \geq \vartheta$

Cost function: $\sum_{v_i \text{ non-leaf in } T} p_i$

Objective: Minimize.

Approx.: Approximable within $O(((k+1)n)^{1/\epsilon}|D|^{2/\epsilon} + kn^2$ [86].

Hardness: NP-hard [86].

Comment: When every node is equipped with the same type of battery, an approximation algorithm has a better performance ratio than that in the general case setting, and the algorithm takes $O(kn|D|\log|D|)$ time [86].

3.4 Restricted Minimum-Energy Broadcast Problem

Instance: A 4-tuple (G, s, d, K) where $G = (V, E)$ is a simple graph, $s \in V$ is the source node, $d < |V|, K < |V|^2$ are positive integers.

Solution: A spanning broadcast tree rooted at s.

Cost function: Total energy in which each transmission radius is at most d.

Objective: Minimize energy, at most K

Hardness: NP-hard [44].

Comment: Proof of NP-completeness of the Restricted Minimum-Energy Broadcast (RMEB) based on reduction from vertex cover problem to RMEB [44].

3.5 Unrestricted Minimum-Energy Broadcast Problem

Instance: A 3-tuple (G, s, K) where $G = (V, E)$ is a simple graph, $s \in V$ is the source node, $K < |V|^2$ is a positive integer.

SOLUTION: A spanning broadcast tree rooted at s.

COST FUNCTION: Total energy

OBJECTIVE: Minimize energy, at most K

Approx.: Optimal solution can be found within $O(n^{k+2})$[44]

3.6 Minimum Broadcast Cover

INSTANCE: A directed graph $G = (V, E)$, a set P consisting of all power levels at which a node can transmit, edge costs $c_{ij} : E(G) \to R_+$, a source node $r \in V$, an assignment operation $p_i^v : V(G) \to P$ and some constant $B \in R_+$.

SOLUTION: A node power assignment vector $A = [p_1^v, p_2^v ... p_{|V|}^v]$ inducing a directed graph $G' = (V, E')$, where $E' = \{(i, j) \in E : c_{ij} \leqslant p_i^v\}$, in which there is a path from r to any node of V (all nodes are covered)

COST FUNCTION: $\sum_{i \in V} p_i^v$

OBJECTIVE: Minimize cost, at most B

Approx.: There exists approximation algorithm that achieves the $O(\log N)$ approximation ratio [22].

Hardness: NP-complete [22].

Comment: There exists an approximation algorithm for the general version which achieves approximation ratio of $18 \log N$.

3.7 Minimum-Energy Broadcast Problem in Multi-hop Wireless Networks

INSTANCE: A wireless ad hoc network $M = (N, L)$, a source node s, and a terminal set $D = N - \{s\}$.

SOLUTION: Broadcast a message from any source node to all the other nodes.

COST FUNCTION: Sum of transmission powers at all nodes.

OBJECTIVE: Minimize.

Approx.: For any source s, approximable within $2H(n-1)$ [94].

Hardness: NP-hard [94].

3.8 Quality of Service Multicast Tree

INSTANCE: Graph $G = (V, E, l, r)$, the length function on each edge $l : E \to R_+$, the rate function on each node $r : V \to R_+$, source s, sets S_i of terminals with rate r_i

SOLUTION: A tree $T = (V_T, E_T, l, r)$ spanning all terminals.

COST FUNCTION: $\sum_{e \in E_T} l(e) r_e$, where $r_e = max(r_i, r_j)$.

OBJECTIVE: Minimize.

Approx.: Approximable within 3.802 [72]

Hardness: NP-hard to within an approximation ratio 96/95 [36].

Comment: Approximable within 1.960 for two non-zero rates.

3.9 Min Power Strong Connectivity

INSTANCE: Directed graph $G = (V, E)$, cost function $c : E \to R_+$

SOLUTION: Strongly connected spanning subgraph H of G

COST FUNCTION: $p(H) = \sum_{u \in V} p_H(u)$, where $p_H(u) = \max\{c(u,v) | (u,v) \in H\}$

OBJECTIVE: Minimize

Approx.: Approximable within approximation ratio 2. Approximable within approximation ratio 1.85 provided G is bidirected [23].

3.10 Min Power Symmetric Connectivity

INSTANCE: Graph $G = (V, E, c)$, cost function $c : E \to R_+$, transmission range function $r : V \to \mathbb{R}^+$, some constant $k \geq 1$.

SOLUTION: A connected graph $T = (V, E_T, c)$ s.t. $r(e_1) \geq c(e)$ and $r(e_2) \geq c(e)$, $e_1, e_2 \in V$.

COST FUNCTION: $\sum_{v \in V} r(v)^k$.

OBJECTIVE: Minimize.

Approx.: Approximable within $5/3 + \epsilon$ for every $\epsilon > 0$ [95] [5].

Hardness: NP-hard for geometric instances in $\mathbb{R}^2$ [37] and APX-complete for instances in $\mathbb{R}^3$ [37].

Comment: More practical approximation algorithm exist with approximation ratio 11/6 [5] , [108].

A variant called Min Power Symmetric Connectivity with Asymmetric Power Requirements is NP-hard to approximate within $(1 - \epsilon) \ln |V|$ [5].

Min Power Symmetric Unicast is efficiently solvable in time $O(|E| \log |V|)$ [5]

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